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Distributional Effects of Surging Housing Costs under Schwabe's Law*

Volker Grossmann†, Benjamin Larin ‡, Hans Torben Löfflad§, Thomas Steger¶

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Abstract

The upward sloping trend of rents and house prices has initiated a debate on the consequences of surging housing costs for wealth inequality and welfare. We employ a frictionless two-sectoral macroeconomic model with a housing sector to investigate the dynamics of wealth inequality and the determinants of welfare. Households have non-homothetic preferences, implying that the poor choose a higher housing expenditure share, which is compatible with Schwabe's Law. We first examine the isolated effects of increasing housing costs in partial equilibrium. The model is closed by introducing a production sector that enables us to analyze the general equilibrium consequences of a widely discussed policy option, which aims at dampening the growth of housing costs. Abolishing zoning regulations triggers a slower rent growth and reduces wealth inequality by 0.7 percentage points (measured by the top 10 percent share). Average welfare increases by 0.5 percent. The household-specific welfare effects are asymmetric. The poor benefit more than the rich, and the richest wealth decile is even worse off.

Key words: Macroeconomics and Housing; Long-Term Growth; Schwabe's Law; Wealth Inequality; Welfare.

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1 Introduction

Since WW2 real housing rents and real house prices have risen, on average, in most industrialized economies (Knoll 2017; Knoll, Schularick, and Steger 2017). At the same time, housing expenditures exhibit a striking pattern. For the US economy, the aggregate housing expenditure share, being the largest single expenditure category, appears largely constant over time at about 19 percent, despite rising real per capita incomes (U.S. Bureau of Labor Statistics 2016; Piazzesi and Schneider 2016). However, at a given point in time, the percentage of total expenditure on housing varies inversely with income. For instance, US households in the first income quintile devoted about 25 percent of their total expenditure to housing in 2015, whereas this number was only 18 percent for the fifth income quintile.[1] This observation is closely related to a pattern that is extremely robust across time and space: Schwabe's law (Singer 1937; Stigler 1954). Indeed, Stigler (1954, p. 100) characterized this as the second fundamental law of consumer behavior.[2]

> *Hermann Schwabe, the director of the Berlin statistical bureau, proposed a second "law" in 1868. He had salary and rent data for 4,281 public employees receiving less than 1,000 thaler a year, and income and rent data for 9,741 families with incomes in excess of 1,000 thaler. For each group he found the percentage of income (or salary) spent on rent declined as income rose, and proposed the law: 'The poorer any one is, the greater the amount relative to his income that he must spend for housing.' The law seemed to contemporaries less obviously true than Engel's, and a considerable literature arose about it. Ernst Hasse found that it held for Leipzig in 1875, and E. Laspeyres confirmed it for Hamburg. Engel also accepted Schwabe's law.*

Surging housing costs under asymmetric spending patterns for housing across income groups have initiated a debate on the implications for wealth inequality and welfare (Summers 2014; Albouy, Ehrlich, and Liu 2016; Albouy and Ehrlich 2018; Dustmann, Fitzenberger, and Zimmermann 2018). We investigate the dynamics of wealth inequality and the determinants of welfare in a growing economy that experiences surging housing costs.[3] Specifically, our analysis addresses *two research questions.* (1) How do the

[1] The cross-sectional variation of housing expenditure shares is even more pronounced in other advanced economies, such as France, Germany, and the UK (Section 6.1).

[2] The first law of consumer behavior is the well-known Engel's law (Stigler 1954).

[3] Housing costs are either user cost of housing (in the case of homeowners) or rents (in the case of

dynamics in real rents interact with (i) wealth inequality and (ii) welfare in a growing economy? (2) How do these interactions depend on Schwabe's law?[4] We first examine the isolated effects of exogenously increasing rents in partial equilibrium. This step is helpful for our general equilibrium analysis which features endogenous rent growth. As a natural candidate for an exogenous event that triggers changes in the time path of rents, we consider the abolishment of zoning regulations. In fact, zoning regulations are widely recognized as an important amplifier of surging rents in a growing economy (Glaeser, Gyourko, and Saks 2005; Saiz 2010; Albouy and Ehrlich 2018).

We employ a frictionless dynamic general equilibrium model with a housing sector. Abstracting from financial frictions enables us to derive analytical insights into the dynamics of wealth inequality and the determinants of welfare. Our analysis captures the systematic impact of future expected rent growth on the saving decisions of forward-looking households. The supply side of the model, which is introduced to endogenize rents, follows the long-term macro and housing model of Grossmann and Steger (2017). It distinguishes between the extensive margin (the number of houses) and the intensive margin (the size of the average house) of the housing stock. This model structure lends itself to investigating the consequences of removing those policies that regulate the use of land for residential purposes and, therefore, primarily constrain the extensive margin of the housing stock. Households are heterogeneous with respect to initial wealth and labor income (Chatterjee 1994; Caselli and Ventura 2000). The demand side features non-homothetic preferences so as to replicate the inverse variation of housing expenditure shares across income groups (Schwabe's law). Specifically, we assume that households have status concerns with respect to housing, which is in line with empirical evidence (Leguizamon and Ross 2012; Bellet 2017).

The analysis proceeds in two steps. In the first, we investigate the dynamics of wealth inequality and the determinants of welfare in partial equilibrium. It is shown that stronger rent growth produces *less* wealth inequality in partial equilibrium, provided that

renters). The major part of the paper is framed in terms of renter households. Nonetheless, our analysis applies equally to homeowners as well as renters (Section 7).

[4] Heterogeneity of housing expenditures as percentage of total consumption expenditures across income groups, which turns out to be directly relevant for welfare, is closely related to Schwabe's law, which focuses on housing expenditures relative to income (Section 2).

the utility function is sufficiently concave. The reason is that the differences in the saving rates across wealth groups (a force contributing to diverging wealth holdings in the population) shrinks in response to stronger rent growth. This counterintuitive result appears to be robust across a large set of models, as it depends merely on the widely accepted assumptions of forward-looking and optimizing households. The analysis also indicates, somewhat surprisingly at first glance, that Schwabe's law is not important with regard to the dynamics of wealth inequality. This insight is in striking contrast to the welfare implications. Stronger status concerns regarding housing induce greater heterogeneity in housing expenditure shares. This amplifies the welfare differences by enlarging the heterogeneity in household-specific price indices. That is, Schwabe's law is important with regard to welfare inequality. The underlying mechanism appears empirically plausible. For instance, Albouy, Ehrlich, and Liu (2016) show that real income inequality in the US increased 25 percent more since 1970, when deflated with household-specific price indices.

In a second step, we analyze a growing economy in general equilibrium. It is shown that the wealth distribution is stationary in a steady state, despite continuously rising housing costs. However, any policy that induces transitional dynamics triggers a permanent change in the wealth distribution. We show that removing residential zoning regulations leads to a temporarily slower rent growth, relative to the baseline scenario, which is associated with a reduction in the top 10 percent wealth share by 0.7 percentage points over time. That is, in contrast to the partial equilibrium result, rent growth and wealth inequality are *positively* associated, i.e. slower rent growth induced by abolishing zoning regulations goes hand in hand with a reduction in wealth inequality. Average welfare increases by about 0.5 percent. However, the household-specific welfare effects are asymmetric, so that the poor benefit more than the rich. The richest wealth decile is even worse off, while welfare of the poorest wealth decile increases by 1.3 percent. The important lessons to be drawn from this policy experiment are twofold. First, despite a potentially negative causal effect of surging rents on wealth inequality, a policy measure that slows down rent growth may nevertheless lower welfare inequality through its additional general equilibrium effects. Second, under Schwabe's law, surging rents are unambiguously and positively associated with welfare inequality and harmful for the poor but not necessarily for the rich.

There are three strands of related literature, the first of which addresses the importance of the housing sector in macro models. Many models are designed to discuss business cycle phenomena, such as Davis and Heathcote (2005), Iacoviello (2005), Iacoviello and Neri (2010), Kiyotaki, Michaelides, and Nikolov (2011), Favilukis, Ludvigson, and Van Nieuwerburgh (2017), and Kydland, Rupert, and Šustek (2016).[5] More recently, a literature has emerged that focusses on the long term, such as Grossmann and Steger (2017), Miles and Sefton (2017) and Borri and Reichlin (2018). Our research questions necessarily require a long-term perspective. The second strand of literature analyzes one-sector economies under household heterogeneity with the representative household property (Chatterjee 1994; Krusell and Rios-Rull 1999; Caselli and Ventura 2000; Alvarez-Pelaez and Díaz 2005; Garcia-Penalosa and Turnovsky 2006). We add to this literature by analyzing a two-sectoral model, allowing for a continuous relative price change, under household heterogeneity with non-homothetic preferences and the representative household property. A third strand examines savings behavior and wealth inequality, by employing stochastic overlapping generations models under incomplete markets. These contributions typically focus on alternative mechanisms that shape the wealth distribution in steady state.[6] We explore mechanisms that shape the dynamics of wealth inequality and welfare differences, apart from borrowing constraints, and provide analytical insights that apply equally to transitional dynamics and the steady state. Our analysis rests on fundamental market forces that would prevail equally in an economy with borrowing constraints, which we ignore for the sake of analytical results.

The structure of this paper is as follows. Section 2 describes the household side. Section 3 provides partial equilibrium results. Section 4 discusses analytical insights into the dynamics of wealth inequality and the determinants of household-specific welfare. Section 5 introduces the production side and characterizes the steady state. Section 6 investigates the consequences of removing zoning regulations numerically in general

[5] Piazzesi and Schneider (2016) provide an excellent survey.

[6] Some exceptions analyze the wealth distribution over time by employing numerical techniques, such as Gabaix et al. (2016), Kaas et al. (2017), Kaymak and Poschke (2016), Hubmer, Krusell, and Smith Jr (2016), and Wälde (2016). De Nardi and Fella (2017) provide an excellent survey.

equilibrium. Section 7 discusses whether the results depend on modelling households as renters or as homeowners.

2 Households

Consider a set of infinitely lived households in a perfectly competitive economic environment. There are $J \in \mathbb{N}$ groups of households indexed by $j \in \{1, 2, \ldots, J\}$. Every group j has measure $n_j \in \mathbb{R}^+$ of households. Each household of group j has time-invariant labor endowment, denoted as l_j, which is supplied inelastically to the labor market.[7] Aggregate labor supply accordingly reads as $L \equiv \sum_j n_j l_j$. The total amount of households is $\mathcal{L} \equiv \sum_j n_j$. Time is continuous and indexed by $t \geq 0$.[8] In addition to labor endowment, l_j, households may also differ in their initial (non-human) wealth holding, $W_j(0)$. Aggregate (non-human) wealth is defined by $W \equiv \sum_j n_j W_j$.

Let c_j denote consumption of the numeraire good of household j, s_j consumption of housing services, and $\bar{s} \equiv \frac{1}{\mathcal{L}} \sum_j n_j s_j$ the average amount of housing services across all households, respectively. Preferences of household j are captured by the intertemporal utility function

$$U_j(0) = \int_0^\infty u(c_j(t), s_j(t)) \mathrm{e}^{-\rho t} \mathrm{d}t \quad \text{with} \tag{1}$$

$$u(c_j, s_j) = \frac{[(c_j)^{1-\theta}(s_j - \phi\bar{s})^\theta]^{1-\sigma} - 1}{1-\sigma}, \tag{2}$$

where $\sigma > 0$ denotes a concavity parameter of the outer utility function, $\theta \in (0, 1)$ a concavity parameter of the inner utility function, $\rho > 0$ the subjective discount rate, and $\phi \in [0, 1)$, respectively. For $\phi > 0$ the utility function (2) captures status concerns with respect to housing services consumption and implies that preferences are non-homothetic. The importance of status concerns is increasing in ϕ.

There are two reasons why we capture status concerns with respect to housing services. First, the importance of status concerns with respect to housing is widely recognized. For instance, by employing US microdata, Bellet (2017) shows that suburban homeowners

[7] In what follows, we employ the short formulation "household j" instead of "household of group j".

[8] The time index t is often suppressed provided that this does not lead to ambiguity.

who experienced a relative downscaling of their homes due to the building of larger units in their suburb record lower satisfaction and house values.[9] Status concerns with respect to housing represent actually an old topic that has already been discussed by Marx (1847):

> *A house may be large or small; as long as the neighboring houses are likewise small, it satisfies all social requirement for a residence. But let there arise next to the little house a palace, and the little house shrinks to a hut. The little house now makes it clear that its inmate has no social position at all to maintain, or but a very insignificant one; and however high it may shoot up in the course of civilization, if the neighboring palace rises in equal or even in greater measure, the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.*

Second, the assumption of status preferences for housing implies, as shown below, that housing expenditure shares, $e_j \equiv \frac{ps_j}{c_j+ps_j}$, differ across households, where p is the price for residential services, i.e. the rent. Specifically, utility function (2) implies that, at a given point in time, the percentage of total expenditures spent on housing, e_j, declines with income (overall wealth).[10] Recall that *Schwabe's law* states that the percentage of income spent on housing, $\frac{ps_j}{y_j}$, declines with income, y_j (as described in Section 1). The two preceding statements are not identical but are related. This becomes obvious by writing $\frac{c_j+ps_j}{y_j} \cdot e_j = \frac{ps_j}{y_j}$. If the poor choose a higher consumption rate ($\frac{c_j+ps_j}{y_j}$), as is empirically plausible and in line with our calibration, then the negative variation of $\frac{ps_j}{y_j}$ with income is even more pronounced than the negative variation of e_j with income (see also Singer 1937).[11]

Each household j chooses consumption paths $\{c_j(t), s_j(t)\}_{t=0}^{\infty}$ by maximizing U_j subject to the standard No-Ponzi-game condition and the intertemporal budget constraint[12]

$$\dot{W}_j = rW_j + wl_j - c_j - ps_j, \tag{3}$$

[9] See also Frank (2005), Turnbull, Dombrow, and Sirmans (2006), and Leguizamon and Ross (2012).

[10] In what follows, we focus on e_j because this variable determines differences in household specific price indices (Section 3).

[11] In Online-Appendix 10.3 we discuss alternative utility specifications. It is shown that the results are robust to assuming status concerns for both goods, provided they are stronger for housing. We also show that alternative formulations (CES utility and multiplicative reference level) are inconsistent with major empirical observations.

[12] A dot above a variable denotes the partial derivative with respect to time.

where r denotes the interest rate and w the wage rate per unit of labor, respectively. As individuals have mass zero, they take factor prices, the rental rate p, and average consumption of housing services, $\bar{s}$, as given. Notice that we model all households as renters. In Section 7 we show that the results do not change if we model households as homeowners instead of renters.

Despite non-homothetic preferences a representative household exists. The following remark makes this property explicit.

Remark 1 (Representative household). *An economy populated by a set of households whose preferences are described by (1) together with (2) and whose intertemporal budget constraint is given by (3) admits a (positive) representative household. That is, the demand side can be described as if there were a single household who owns the entire endowments and makes the aggregate consumption and saving decisions. As a consequence, the distribution of labor endowment, l_j, and wealth, W_j, does not play a role for the evolution of aggregate variables. This does also apply for the case of non-homothetic preferences ($\phi > 0$).*

The validity can be proven by showing that the aggregation of the household's first-order conditions yields the same set of first-order conditions that result from the problem of a single household who owns the entire endowments, L and $W(0)$, and is making the aggregate consumption and saving decisions. All proofs are relegated to the Appendix.

The representative household setup surely has costs and benefits. On the one hand, it abstracts from feedback effects of a changing wealth and income distribution on aggregate variables. On the other hand, this setup enables us to gain analytical insights into the dynamics of wealth inequality and the determinants of welfare. In fact, abstracting from the feedback effects of distributional changes on aggregate variables, including prices, is precisely the reason why we are able to derive analytical insights, as shown below. Our paper, therefore, investigates those fundamental mechanisms that operate even in the absence of financial market frictions.[13]

[13] We add further to the theoretical literature on dynamic macro models with a representative household by analyzing a two-sectoral model under heterogeneity and non-homothetic preferences. Previous macro models under household heterogeneity with the representative household property are mostly, if not exclusively, one-sectoral models (Chatterjee 1994; Krusell and Rios-Rull 1999; Caselli and Ventura 2000;

3 Household Behavior

This section characterizes household behavior. All prices, $\{r, w, p\}$, are taken as given at this layer of analysis. The propositions described below are important when it comes to understanding the wealth and welfare implications of surging rents, which will be investigated, first, in partial equilibrium (Section 4) and, second, in general equilibrium (Section 5 & 6) below.

3.1 Housing Expenditure Shares

Define total wealth of household j, $\mathcal{W}_j$, as the sum of its non-human wealth, W_j, and human wealth, $\tilde{w}l_j$

$$\mathcal{W}_j(t) \equiv W_j(t) + \tilde{w}(t)l_j \quad \text{with} \tag{4}$$

$$\tilde{w}(t) \equiv \int_t^\infty w(\tau)\mathrm{e}^{\int_t^\tau -r(v)\mathrm{d}v}\mathrm{d}\tau. \tag{5}$$

Let $\bar{W}$ denote average non-human wealth in the economy and define the average labor supply by $\bar{l} \equiv L/\mathcal{L}$. Average total wealth is thus given by $\bar{\mathcal{W}} \equiv \bar{W} + \tilde{w}\bar{l}$. Also define the relative to average total wealth level of household j by $\Omega_j \equiv \mathcal{W}_j/\bar{\mathcal{W}}$.

Proposition 1 (Housing expenditure shares) *The expenditure share for housing services of household j, defined as $e_j \equiv \frac{ps_j}{c_j+ps_j}$, is time invariant and given by*

$$e_j = \theta\left(1 + \frac{(1-\theta)\phi}{[1-(1-\theta)\phi]\,\Omega_j(0)}\right). \tag{6}$$

For $\phi > 0$, there is a negative relationship between e_j and the relative total wealth level of household j in the initial period, $\Omega_j(0)$.

The housing expenditure share of the representative household, with $\Omega_j = 1$, is given by $\bar{e} \equiv \frac{\theta}{1-(1-\theta)\phi}$. Most importantly, the housing expenditure share is decreasing in total relative wealth at time $t = 0$ whenever individuals have status preferences ($\phi > 0$).

Alvarez-Pelaez and Díaz 2005; Garcia-Penalosa and Turnovsky 2006).

3.2 Ideal Price Indices

Instantaneous utility may be written as $u(\mathcal{C}_j) \equiv \frac{(\mathcal{C}_j)^{1-\sigma}-1}{1-\sigma}$ with consumption index $\mathcal{C}_j \equiv (c_j)^{1-\theta}(s_j - \phi\bar{s})^{\theta}$. The associated overall consumption expenditure is denoted as $\mathcal{E}_j \equiv c_j + ps_j$. Hence, the price index of a household j is given by $\mathcal{P}_j \equiv \mathcal{E}_j/\mathcal{C}_j$. When $\mathcal{C}_j$ and $\mathcal{P}_j$ are evaluated at equilibrium quantities and prices, we refer to $\mathcal{C}_j$ and $\mathcal{P}_j$ as ideal consumption index and ideal price index of j.

Proposition 2 (Ideal price indices) *The ideal price index of household j in period t is given by*

$$\mathcal{P}_j(t) = \frac{p(t)^{\theta}}{\theta^{\theta}(1-\theta)^{-\theta}} \frac{1}{1-e_j} \equiv \tilde{\mathcal{P}}(p(t), e_j). \tag{7}$$

The ideal price index of the representative household (equal to the aggregate price index) is obtained for $e_j = \bar{e}$. If $\phi = 0$ (homothetic preferences), i.e. $e_j = \theta$ for all j, the price index is the same for all households and given by $\tilde{\mathcal{P}}(p, \theta) = \frac{p^{\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}}$. For $\phi > 0$, we obtain partial derivatives $\frac{\partial \tilde{\mathcal{P}}(p,e)}{\partial e} > 0$ and $\frac{\partial \tilde{\mathcal{P}}(p,e)}{\partial e \partial p} > 0$. That is, the poorer a household is in terms of its total wealth, i.e. the higher the housing expenditure share (Proposition 1), the stronger is $\mathcal{P}_j$ affected by an increase in the price for housing services, p.[14]

3.3 Saving Rates

Let $y_j \equiv rW_j + wl_j$ denote income and let $\mu_j \equiv \mathcal{E}_j/\mathcal{W}_j$ denote the (average) propensity to consume out of total wealth.[15] The saving rate $sav_j = 1 - \mathcal{E}_j/y_j$ will turn out being helpful in the subsequent analysis. To discuss the properties of this saving function, we turn first to the propensity to consume.

[14] Albouy, Ehrlich, and Liu (2016) construct an ideal cost-of-living index that varies with income and prices. They show that, based on US microdata, real income inequality, measured by the 90 percentile / 10 percentile ratio, rose by 10 percentage points more when income is deflated by their individual cost-of-living index.

[15] In our model, the average propensity to consume equals the marginal propensity to consume.

Proposition 3 (Propensity to consume) *The propensity to consume is at any time t identical for all households and given by*

$$\mu(t) = \left(\int_t^\infty \left(\frac{\bar{p}(\tau,t)^\theta}{\exp\left[\bar{r}(\tau,t) + \frac{\rho}{\sigma-1}(\tau - t)\right]} \right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\tau \right)^{-1}, \tag{8}$$

where $\bar{r}(\tau,t) \equiv \int_t^\tau r(v)\mathrm{d}v$ is the cumulative interest rate and $\bar{p}(\tau,t) \equiv p(\tau)/p(t)$ is the growth factor of the housing rent between τ and t.

That all households choose the same propensity to consume reflects the absence of heterogeneity in terms of preferences and the absence of borrowing constraints. At next we turn to the saving function.

Proposition 4 (Saving rates) *Let $\omega_j = W_j/l_j$ denote the wealth-to-labor ratio of household j. Its saving rate at any time t may be expressed as*

$$sav_j(t) = 1 - \frac{\mu(t) \cdot [\omega_j(t) + \tilde{w}(t)]}{r(t)\omega_j(t) + w(t)} \equiv Sav(\omega_j(t), \cdot). \tag{9}$$

The notation $sav_j = Sav(\omega_j, \cdot)$ highlights that the saving rate is a function of the wealth-to-labor ratio, ω_j. An implication of the preceding proposition is given by

Corollary 1 (Saving rate differentials). *The saving rate changes with the wealth-to-labor ratio according to $\frac{\partial Sav_j}{\partial \omega_j} = \frac{\mu(r\tilde{w}-w)}{(\omega_j r+w)^2}$. That is, the saving rate increases with ω_j, $\frac{\partial Sav_j}{\partial \omega_j} > 0$, provided that $r\tilde{w} > w$.*

To understand Corollary 1 notice that the ratio of total wealth to income, $\mathcal{W}_j/y_j$, can be expressed as

$$\frac{\mathcal{W}_j}{y_j} = \frac{W_j + \tilde{w}l_j}{rW_j + wl_j} = \frac{1}{r}\frac{r\omega_j}{r\omega_j + w} + \frac{\tilde{w}}{w}\frac{w}{r\omega_j + w}. \tag{10}$$

The term $1/r$ represents the total-wealth-to-income ratio of a pure capitalist ($W_j > 0$, $l_j = 0$), while the term $\tilde{w}/w$ represents the total-wealth-to-income ratio of a pure worker ($W_j = 0$, $l_j > 0$). The condition $\tilde{w}/w > 1/r$ (or $r\tilde{w} > w$) is thus equivalent to assuming that ratio $\mathcal{W}_j/y_j$ of a pure worker exceeds that of a pure capitalist. In this case, for given factor prices, $\mathcal{W}_j/y_j$ is decreasing in ω_j. Hence, households characterized by a low ω_j choose a high level of consumption relative to income, $\mu\mathcal{W}_j/y_j$, implying that the saving rate, $sav_j = 1 - \mu\mathcal{W}_j/y_j$, is low, and vice versa.

Notice also that the condition $r\tilde{w} > w$ is satisfied in any steady state with positive wage growth. Assuming $w(\tau) = w(t)e^{(\tau-t)g}$ with $g > 0$, we have $\tilde{w}(t) = \frac{w(t)}{r-g}$. Plugging this into $r\tilde{w}(t) > w(t)$ boils down to $g > 0$.

4 Wealth and Welfare in Partial Equilibrium

We are now ready to discuss the dynamics of wealth inequality and the determinants of welfare analytically in partial equilibrium, i.e. taking prices as exogenous. The mechanisms discussed below still hold in general equilibrium, when prices are fully endogenous.

4.1 Surging Rents and Wealth Inequality

4.1.1 Wealth Divergence and Wealth Convergence

Let the growth rate of (non-human) wealth of household j be defined as $\hat{W}_j \equiv \dot{W}_j/W_j$ and express the saving rate as $sav_j \equiv \dot{W}_j/y_j$. The growth rate of household j's wealth may then be expressed as follows:[16]

$$\hat{W}_j \equiv \frac{\dot{W}_j}{W_j} = sav_j \frac{y_j}{W_j} = Sav(\omega_j, \cdot)\left(r + \frac{w}{\omega_j}\right) \equiv G(\omega_j, \cdot), \tag{11}$$

where we used $y_j = rW_j + wl_j$ for income and $\omega_j = W_j/l_j$. To simplify the analysis, we assume

Assumption 1. *If $W_j \geq W_{j'}$ for any $j \neq j'$, then it also holds true that $\omega_j \geq \omega_{j'}$.*

Notice that $\omega_j \geq \omega_{j'}$ implies $W_j/W_{j'} \geq l_j/l_{j'}$. That is, relative wealth between any two households j and j' is not smaller than relative earnings. This assumption does

[16] The notation $G(\omega_j, \cdot) = Sav(\omega_j, \cdot)\left(r + \frac{w}{\omega_j}\right)$ highlights that the wealth growth rate is a function of $\omega_j = W_j/l_j$. The function $G(\omega_j, \cdot)$ is well defined for positive and negative wealth. It is not defined for $W_j = 0$. However, the limits are defined: $\lim_{W_j \to 0-} \hat{W}_j = -\infty$ and $\lim_{W_j \to 0+} \hat{W}_j = +\infty$.

not seem too restrictive, given that wealth is more unequally distributed than earnings (Kuhn and Ríos-Rull 2016). With Assumption 1 at hand, we define wealth divergence and wealth convergence as follows. Suppose $\omega_j > \omega_{j'}$. Wealth levels diverge (converge) between j and j' in a small time interval $[t, t + \mathrm{d}t]$ if $\hat{W}_j(t) > (<) \hat{W}_{j'}(t)$.[17] If this is true for any two households, there is *global wealth divergence* (*convergence*). Equation (11) is instructive for two reasons. First, it represents a general statement that is not conditional on any specific model. Second, it indicates that there are two opposing forces at work. On the one hand, wealth-rich households choose a higher saving rate compared to the wealth poor, provided that $r\tilde{w} > w$ (Corollary 1). This represents a divergence mechanism. On the other hand, the income-to-wealth ratio, $\frac{y_j}{W_j} = r + \frac{w}{\omega_j}$, is unambiguously decreasing in ω_j. This represents a convergence mechanism.

Substituting (9) into (11), one obtains

$$\hat{W}_j = G(\omega_j, \cdot) = r - \mu + \frac{w - \mu\tilde{w}}{\omega_j}. \tag{12}$$

Thus, the wealth distribution is stationary in any period t provided that $\mu(t)\tilde{w}(t) = w(t)$ holds.[18] We will show that this condition holds in the steady state. It means that consumption out of human wealth equals contemporaneous earnings. As a result, the growth rate of wealth is the same across households and equal to $\hat{W}_j = r - \mu$, according to (12). That is, in view of (11), the condition $\mu\tilde{w} = w$ ensures that the divergence mechanism, $\frac{\partial Sav(\omega_j, \cdot)}{\partial \omega_j} > 0$, compensates the convergence mechanism, $\frac{\partial (r + w/\omega_j)}{\partial \omega_j} < 0$.

An overall measure for the change of the wealth distribution, summarizing the net effect of the divergence and the convergence mechanism, is given by the derivative $\frac{\partial G(\omega_j, \cdot)}{\partial \omega_j} = \frac{\mu\tilde{w} - w}{(\omega_j)^2}$. It shows that there is wealth divergence (convergence) in period t for $\mu(t)\tilde{w}(t) < (>) w(t)$, i.e. when $\frac{\partial G(\omega_j, \cdot)}{\partial \omega_j} > (<) 0$.

[17] In the following, rather than referring to a change within time interval $[t, t + \mathrm{d}t]$ we will refer to a change at point in time t.

[18] Recall that the distribution of wealth (holding the mean constant) does not affect aggregate quantities and prices (Remark 1).

4.1.2 The Rent Channel

We are now ready to discuss how rising real rents, holding $\{r, w\}$ constant, affect wealth inequality.

Proposition 5 (Rent channel) *An increase (decrease) in the growth of real rents between the current period t and some future period τ, measured by $\bar{p}(\tau, t)$, contributes to less (more) wealth inequality in the current period t, measured by $\frac{\partial G(\omega_j(t),\cdot)}{\partial \omega_j(t)}$, if $\sigma > 1$ ($\sigma < 1$).*

Let us focus on the empirically relevant case of a sufficiently concave utility function ($\sigma > 1$).[19] Stronger rent growth, measured by an increase in $\bar{p}(\tau, t)$, induces less wealth inequality. The economic intuition behind this result can be explained in two steps. First, all households choose a higher saving rate (by reducing the propensity to consume) to provide for the future rent burden in order to smooth housing consumption over time. This results immediately from Propositions 3 and 4. Second, this increase in the saving rates is asymmetric across households. It is stronger for the wealth poor than for the wealth rich. To see this, notice that the saving rate, $sav_j = 1 - \mu \mathcal{W}_j / y_j$, increases as μ is being reduced. Recall that the total-wealth-to-income ratio, $\mathcal{W}_j / y_j$ is decreasing in ω_j, and thus decreasing in wealth level W_j (Assumption 1), provided that $r\tilde{w} > w$. Thus, the wealth-poor exhibit a comparably high total-wealth-to-income ratio, $\mathcal{W}_j / y_j$, implying that a reduction in the propensity to consume in response to an increase in the growth of rents implies a comparably strong increase in the saving rate. As a result, the differences in the saving rates across wealth groups are being reduced and the divergence mechanism is weakened.

This dampening effect of rising rents on wealth inequality depends on the assumptions of forward-looking and optimizing households together with an empirically plausible intertemporal elasticity of substitution ($\frac{1}{\sigma} < 1$). It can, therefore, be expected to be robust across a large set of different models.[20] The analysis also clarifies that accounting for

[19] The calibration is explained in Section 6.1.

[20] As a caveat, if the poor cannot finance going consumption expenditures by running into debt, their propensity to consume is lower compared to the unconstrained case. Stronger rent growth may then not allow these households to lower their propensity to consume and increase their saving rate.

Schwabe's law does not matter in this context.[21]

4.2 Status Concerns, Price Indices, and Welfare

How does the status-induced heterogeneity of housing expenditure shares affect the distribution of household-specific welfare in a growing economy? To discuss this question, we consider the welfare position of household j relative to the representative household. Specifically, we ask by how much household j is better off, in terms of consumption-equivalent variations, relative to the representative household. Denoting by $\bar{\mathcal{C}}(\tau)$ the ideal consumption index of the representative agent at time $\tau \geq t$, welfare measure $\psi_j(t)$ equals the percentage consumption increase that must be given to the representative household such that he / she is equally well of as j:

$$\int_t^\infty \frac{\left[(1+\psi_j)\,\bar{\mathcal{C}}(\tau)\right]^{1-\sigma}-1}{1-\sigma}\mathrm{e}^{-\rho(\tau-t)}\mathrm{d}\tau = U_j(t). \tag{13}$$

For the representative household $\psi_j(t) = 0$ by definition. If $\psi_j(t) > (<) 0$, then household j is better (worse) off than the representative household.

Proposition 6 (Welfare) *The welfare of a household j relative to the representative household, at any time t, is given by*

$$\psi_j(t) = \frac{\mathcal{W}_j(t)}{\bar{\mathcal{W}}(t)}\left(\frac{\mathcal{P}_j(t)}{\bar{\mathcal{P}}(t)}\right)^{-1} - 1 = \frac{\Omega_j(t)}{\frac{\mathcal{P}_j(t)}{\bar{\mathcal{P}}(t)}} - 1. \tag{14}$$

Relative welfare of household j, measured by $\psi_j(t)$, depends positively on relative overall wealth, $\Omega_j(t) = \frac{\mathcal{W}_j(t)}{\bar{\mathcal{W}}(t)}$, and negatively on the relative household-specific price index, $\frac{\mathcal{P}_j(t)}{\bar{\mathcal{P}}(t)}$.[22] The first term, $\Omega_j(t) = \frac{\mathcal{W}_j(t)}{\bar{\mathcal{W}}(t)}$, is standard (e.g. Caselli and Ventura 2000).

[21] This is consistent with a two-stage logic. First, households maximize life time utility w.r.t. $\mathcal{C}_j$ (intertemporal problem). Second, households maximize instantaneous utility w.r.t. c_j and s_j (intratemporal problem). Within the setup at hand the decisions at both stages are separable. Moreover, Section 7 shows that the mechanisms discussed above do not depend on whether households are modelled as renters or homeowners.

[22] If the propensities to consume were not the same across agents, then welfare of agent j would equal to welfare of agent j' if we multiply, for all $\tau \geq t$, ideal consumption $\mathcal{C}_{j'}(\tau)$ by $\frac{\mu_j(t)}{\mu_{j'}(t)}\frac{\mathcal{W}_j(t)}{\mathcal{W}_{j'}(t)}\left(\frac{\mathcal{P}_j(t)}{\mathcal{P}_{j'}(t)}\right)^{-1}$. This expression points to an additional channel in models with, say, borrowing constraints that operates via differences in the propensities to consume.

The second term, $\left(\frac{\mathcal{P}_j(t)}{\bar{\mathcal{P}}(t)}\right)^{-1}$, adds a new channel. The relative price index enters because of the two-sectoral structure together with non-homothetic preferences, as can be seen as follows. While the ideal price index is trivially equal to unity in a one-sectoral model, in a two-sectoral model under homothetic preferences ($\phi = 0$) the ideal price index, $\mathcal{P} = \frac{p^\theta}{\theta^\theta (1-\theta)^{1-\theta}}$, is identical across households. It is the combination of a two-sectoral model structure and non-homothetic preferences that gives rise to household-specific price indices as an independent source for welfare differences.

Consider the general case of a two-sectoral economy under non-homothetic preferences. By substituting (6) into (7), one gets

$$\mathcal{P}_j(t) = \frac{p(t)^\theta}{\theta^\theta (1-\theta)^{1-\theta}} \frac{1-(1-\theta)\phi}{1-(1-\theta)\phi - \frac{\theta\phi}{\Omega_j(0)}} \tag{15}$$

for all $t \geq 0$. Thus, for $\phi > 0$, the price index of household j is decreasing with relative total wealth in the initial period, $\Omega_j(0)$. That is, the household-specific price index is relatively large for wealth-poor households. As a consequence, the weak welfare position of a wealth-poor household ($\Omega_j < 1$) for a given price index (like in a one-sector economy) is being further worsened by a price index above the average, i.e. $\mathcal{P}_j(t) > \bar{\mathcal{P}}(t)$.

An important implication of Proposition 6 together with (15), noting that overall wealth Ω_j is time invariant, is given by

Corollary 2 (Amplification of welfare differences). *Stronger status concerns with respect to housing amplify, at any t, the welfare differences, measured by $\psi_j(t)$, i.e.*

$$\frac{\partial \psi_j(t)}{\partial \phi} = \frac{\theta\left[\Omega_j(t) - 1\right]}{(\phi-1)^2} \begin{cases} > 0 & \text{for} \quad \Omega_j(t) > 1 \\ < 0 & \text{for} \quad \Omega_j(t) < 1 \end{cases} . \tag{16}$$

Stronger status concerns with respect to housing improve the relative welfare position $\psi_j(t)$ of wealth rich households, $\Omega_j(t) > 1$, and worsen the relative welfare position $\psi_j(t)$ of wealth poor households, $\Omega_j(t) < 1$.[23] As a result, stronger status concerns amplify the welfare differences across households. The intuition is simple. Stronger status concerns

[23] Strictly speaking, Corollary 2 focuses on the first-order effect of a change in ϕ, neglecting possible feedback effects due to changes of Ω_j that may occur in general equilibrium.

magnify the (endogenous) heterogeneity in housing expenditure share. Hence, the dispersion of household-specific price indices and the welfare distribution is getting more unequal. Stated differently, although Schwabe's law is inconsequential for the effects of higher rent growth on wealth inequality, it has first order welfare implications.

5 General Equilibrium

So far, prices $\{r, w, p\}$ have been taken as given. To endogenize prices, we close the model by introducing the production sector. The definition of the general equilibrium is given in Appendix 9.1.[24]

5.1 Firms

We employ the two-sectoral macro model with a housing sector of Grossmann and Steger (2017) on the production side. This model is designed to think long term and distinguishes between the extensive margin (number of houses) and the intensive margin of the housing stock (size of the average house). Notice that residential zoning regulations, by constraining the economic use of land, affect primarily the number of houses.[25] Because the model distinguishes both dimensions of the housing stock, it is well suited to investigate the macroeconomic implications of zoning regulations.

5.1.1 Numeraire Good Sector

The non-residential sector produces a final good, Y, chosen as numeraire, according to a Cobb-Douglas production function:

$$Y = (K^Y)^{\alpha}(B^Y L^Y)^{\beta}(B^Y Z^Y)^{1-\alpha-\beta}, \tag{17}$$

where K^Y, L^Y and Z^Y denote physical capital, labor and land devoted to the Y sector, respectively. The productivity parameter, $B^Y > 0$, grows exponentially at constant rate

[24] The reduced-form, dynamic system is stated in Online-Appendix 10.1.

[25] Zoning regulations are intended to separate different types of land use (Gyourko and Molloy 2015). We stress that zoning regulations constrain the availability of land for residential purposes.

$g^Y \geq 0$. The technology parameters $\alpha, \beta > 0$ satisfy $\alpha + \beta < 1$. The capital resource constraint reads $K^Y = K$, where $K = \sum_j n_j K_j$ denotes the total supply of physical capital. Capital depreciates at rate $\delta^K \geq 0$ such that gross physical capital investment reads $I^K \equiv \dot{K} + \delta^K K$. $K(0)$ is given.

5.1.2 Housing Sector

There are three types of firms in the housing sector. Real estate development firms invest in infrastructure and transform non-residential land into developed real estates (residential land). Real estate development diminishes the amount of land that can be employed in the Y sector. Overall supply of economically usable land, Z, is exogenous and assumed to be fixed. Housing services firms combine a developed real estate with residential buildings to produce housing services. Construction firms manufacture residential buildings by employing materials and labor.

Real Estate Development The amount of houses is denoted by N, a real number. It captures the extensive margin of the housing stock. $N(0)$ is given. Real estate development firms transform one unit of non-residential land into one unit of residential land. Total land usage in the housing sector is given by $N \leq \kappa Z$, where $0 < \kappa < 1$ denotes a policy parameter that may constrain the amount of residential land. In equilibrium $N + Z^Y = Z$ holds.

Let P^Z denote the price per unit of non-residential land, which is allocated to the numeraire sector. In equilibrium, $P^Z = \int_t^\infty R^Z(\tau) e^{\int_t^\tau -r(v)\mathrm{d}v} \mathrm{d}\tau$, where $R^Z = \partial Y / \partial Z^Y$ denotes the competitive rental rate of non-residential land and r the interest rate, respectively. The costs $\mathfrak{C}(\dot{N}, P^Z, w)$ of increasing the number of developed real estates by $\dot{N}$ amount to

$$\mathfrak{C}(\dot{N}, P^Z, w) = P^Z \dot{N} + w \frac{\xi}{2} (\dot{N})^2, \tag{18}$$

$\xi > 0$. The first cost component, $P^Z \dot{N}$, shows the costs associated with the purchase of $\dot{N}$ units of land. The second component, $w \frac{\xi}{2} (\dot{N})^2$, captures convex adjustment costs for transforming non-residential land into residential land (or vice versa), where w denotes the wage rate and $L^N = \frac{\xi}{2} (\dot{N})^2$ displays the required units of labor to transform $\dot{N}$ units of land.

Housing Services Producing housing services requires to purchase a developed real estate (the fixed input at the level of housing services firm) and combine it with structures (the variable input). The amount of housing services per house produced increases with the amount of residential structures employed per house. However, because a developed real estate serves as fixed input, it increases less than proportionately with the amount of residential structure. That is, the production of housing services per house is characterized by decreasing returns to scale. Formally, let x denote the amount of structures per housing project. An amount x produces housing services h per house according to

$$h = x^{\gamma}, \tag{19}$$

$0 < \gamma < 1$. Total consumption of housing services cannot exceed total supply, i.e. $\sum_j n_j s_j \leq Nh$. Denoting the rental rate of structures by R^X, profits (residual income) from supplying housing services are given by $\pi \equiv ph - R^X x$ per house. Thus, in equilibrium, $R^X = p\gamma x^{\gamma-1}$ and $\pi = (1-\gamma)ph$.

Construction The construction firm produces structures, that are rented out to housing services producers, by combining labor, L^X, and construction materials, M. The production of one unit of materials requires one unit of the numeraire good. Gross investment in structures are produced according to $I^X = M^{\eta}(B^X L^X)^{1-\eta}$, $0 < \eta < 1$, where $B^X > 0$ grows exponentially at constant rate $g^X \geq 0$. The total stock of structure, X, evolves according to

$$\dot{X} = \underbrace{M^{\eta}(B^X L^X)^{1-\eta}}_{=I^X} - \delta^X X, \tag{20}$$

where $\delta^X > 0$ denotes the depreciation rate of residential structure and $X(0)$ is given. The amount of residential buildings that is employed by all housing services firms must satisfy $Nx \leq X$. Construction labor is limited by $L^X \leq L - L^Y - L^N$.

5.2 Wealth

The portfolio of any household j comprises ownership claims on physical capital (K_j), the ownership of housing units (N_j), and non-residential land (Z_j^Y). Total asset holdings per household j therefore read as

$$W_j \equiv \underbrace{P^H N_j}_{\text{housing wealth}} + \underbrace{\overbrace{K_j}^{\text{physical capital}} + \overbrace{P^Z Z_j^Y}^{\text{non-residential land wealth}}}_{\text{non-residential wealth}}, \tag{21}$$

where the house price, $P^H \equiv q^N + q^X x$, is the sum of the value of a developed real estate (q^N) and the value of the employed structure ($q^X x$), where q^X denotes the value per unit of structure.[26]

5.3 Steady State

Steady state growth rates of all variables are linear transformations of the growth rates of productivity parameters, g^X and g^Y. They are stated in Appendix 9.2. Here we focus on two price variables. Denote the steady state growth rate of the rent by $\hat{p}$ and that of the house price by $\hat{P}^H$. It can be shown that

$$\hat{p} = (1 - \gamma\eta)\, g^Y - \gamma\,(1 - \eta)\, g^X \tag{22}$$

and $\hat{P}^H = g^Y$.[27] Also the gross domestic product (GDP) and the wage rate grow in steady state at rate g^Y. Intuitively, higher income growth (increase in g^Y) raises $\hat{p}$ by raising the demand for housing (meeting a fixed long run supply of the number of houses, N), whereas higher productivity growth in the construction sector (increase in g^X) reduces rent growth.

Given that the rent and the house price grow exponentially, one would like to know how the wealth distribution behaves in a steady state. The subsequent proposition clarifies this aspect.

[26] In equilibrium, the value per developed real estate is bid up to the present discounted value (PDV) of operating profits, $q^N(t) = \int_t^\infty \pi(\tau) e^{\int_t^\tau -r(v)\mathrm{d}v} \mathrm{d}\tau$, and $q^X(t) = \int_t^\infty R^X(\tau) e^{\int_t^\tau -(r(v)+\delta^X)\mathrm{d}v} \mathrm{d}\tau$.

[27] See the proof of Proposition A.1 (Steady state) in Appendix 9.3.

Proposition 7 (Stationary wealth distribution) *In a steady state, the wealth distribution is stationary in the sense that, for any two households j and j', the relative wealth position $W_j/W_{j'}$ does not change over time. W_j grows at rate g^Y for all j.*

The proof of Proposition 7 shows that $\mu\tilde{w} = w$ is satisfied in any steady state, implying stationarity of the wealth distribution, according to (12). Consequently, a change in wealth inequality over time requires some form of transitional dynamics. The policy measure analyzed in Section 6.2 triggers such transitional dynamics.

6 Numerical Analysis

We now investigate the consequences of abolishing residential zoning regulations on wealth inequality and welfare in general equilibrium by comparing two scenarios. In the baseline scenario, we consider a steady state with binding zoning regulation. In this scenario the rent grows at constant growth rate, as given by (22), and the wealth distribution is stationary. In the policy-reform scenario, we consider transitional dynamics in response to the counterfactual abolishment of zoning regulations, implying that the rent grows temporarily at a lower pace than in the baseline scenario.[28]

6.1 Calibration

We calibrate the model economy's steady state to the postwar US economy at an annual frequency. This implies a stationary wealth distribution, which is roughly in line with recent data on the wealth distribution (WID 2017; Kuhn, Schularick, and Steins 2018).[29]

Household sector Total population is normalized to unity ($\mathcal{L} = 1$), implying that average and aggregate variables coincide. We calibrate the joint distribution of initial wealth and labor productivity by matching wealth deciles and average earnings of the age group 33-55 from the US Survey of Consumer Finances (SCF) in 2013.[30] Similar to

[28]The numerical solution procedure is described in detail in Online-Appendix 10.2.

[29]The set of parameters is summarized in Appendix 9.4.

[30]See www.federalreserve.gov/econres/scfindex.htm. We are grateful to Moritz Kuhn for providing the data.

Kuhn and Ríos-Rull (2016) and Krusell and Rios-Rull (1999), we focus on this age-group to calibrate a dynastic model by abstracting from life-cycle effects. We consider $J = 10$ wealth groups, in ascending order, that correspond to the observed wealth deciles and the average earnings within each decile.[31] Moreover, Havránek (2015) shows that the majority of studies find an intertemporal elasticity of substitution (IES) below 0.8. We set $\sigma = 2$, implying an IES of 0.5. It is further assumed that every household holds the same portfolio composition as the representative agent.

quintile	aggregate	1st	2nd	3rd	4th	5th
US data	19	25	21	20	19	18
Model: baseline calibration	19	25	22	20	19	18
UK data (normalized)	19	33	23	19	16	15
Model: alternative calibration	19	34	26	23	20	16

Table 1: Housing expenditure shares by income quintiles in percent.

Notes: (a) Housing expenditure share is defined as the ratio of expenditures on housing services (including imputed rent) and total consumption expenditures. (b) First row "U.S. data" shows the empirical values for the US in 2015. Data source: www.bls.gov/cex/data.htm (accessed June 19, 2017). (c) Second row "Model: baseline calibrated" shows the model based expenditure shares such that $\bar{e} = 0.19$ and the difference between first and fifth income quintile according to U.S. data (7 percentage points) is matched. (d) Third row "Model: alternative calibrated" shows the model based expenditure shares such that $\bar{e} = 0.19$ and the difference between first and fifth income quintile of the (to $\bar{e} = 0.19$) normalized distribution according to UK data published by Office for National Statistics (2015) is matched.

The preference parameters ϕ and θ are set to match two key moments of the expenditure share distribution in the US in 2015, as displayed in Table 1: (i) An aggregate housing expenditure share of 19 percent and (ii) a difference between the expenditure shares of the first and fifth income quintiles of 7 percentage points. This results in $\phi = 0.104$ and $\theta = 0.174$.

To study sensitivity, two alternative values for ϕ and θ are considered. When changing ϕ, we adjust θ such that the aggregate housing expenditure share, $\bar{e}$, remains at 19

[31] This implies also that each group is of the same size, $n_j = 1/J = 0.1$.

percent. For a given ϕ this implies, together with the FOC (30), that θ is obtained from

$$\theta = \frac{\bar{e}(1-\phi)}{1-\phi\bar{e}}. \tag{23}$$

First, we consider $\phi = 0$ (no status preferences), implying that housing expenditure shares are homogeneous. This leads to $\theta = 0.19$. Second, the case of strong status preferences is motivated by the case of UK, where the heterogeneity in housing expenditures is considerably higher than in the US.[32] We normalize the UK data to the average US housing expenditure share as can be seen in Table 1. Matching the difference between housing expenditure shares at the first and fifth income quintiles of 18 percentage points yields $\phi = 0.26$ and $\theta = 0.148$.

In a two-sectoral model the steady state growth rate of consumption, as implied by the Euler equation, also depends on the growth rate of the relative price of the two consumption goods, p. Hence, the time preference rate, ρ, has to be calibrated jointly with θ and γ. We match the average rate of return on wealth for the postwar US of 5.77 percent (Jordà et al. 2018, Table 12). This yields $\rho = 0.019$.

Although we do not calibrate the model to match the saving rate, the calibrated model is compatible with empirical observations. Our calibrated model implies that the saving rate of the representative consumer (equal to the aggregate saving rate), $\overline{sav} \equiv \dot{\bar{W}}/\bar{y}$ with income $\bar{y} \equiv r\bar{W} + w\bar{l}$, equals 11.8 percent. This value is in line with the US aggregate saving rate of 9 percent on average from 1950 to 2010 (Piketty and Zucman 2014, Table A86). The saving rates of the 1st to 5th income quintiles are 0.9, 1.8, 4.7, 8.5, and 17.1 percent. These values are in the range of the estimated saving rates by Dynan, Skinner, and Zeldes (2004).

Numeraire sector The total amount of land that can be used economically, Z, is normalized to one. The annual depreciation rate of capital, δ^K, is set to 5.6 percent

[32] The aggregate expenditure share in the UK amounts to 17 percent, while the expenditure shares in income quintiles 1-5 read as {29, 20, 17, 14, 14} (Office for National Statistics, 2015). For Germany (2013) the aggregate expenditure share amounts to 27 percent, while the expenditure shares in income quintiles 1-5 read {37, 33, 29, 27, 24} (Statistisches Bundesamt 2015). The aggregate housing expenditure share in France is 22 percent (averaged over 2011 to 2015), while the expenditure shares of income quintiles 1-5 read {26, 24, 24, 23, 18} (Accardo, Billot, and Buron 2017).

(Davis and Heathcote 2005). The concavity parameters of the production function for the numeraire good, α and β, are set to match the sector's expenditure shares for labor, β, and land, $1-\alpha-\beta$. Grossmann and Steger (2017) compute $\beta = 0.69$ and $1-\alpha-\beta = 0.03$, implying $\alpha = 0.28$. GDP grows at the rate g^Y in the model economy and therefore g^Y is set equal to the average annual growth rate of real US GDP per capita of 2.0 percent between 1950 and 2017.[33]

Housing sector The annual depreciation rate of structures, δ^X, is set to 1.5 percent (Hornstein 2009, p. 13). The labor expenditure share in the construction sector, $1-\eta$, amounts to 62 percent on average in the postwar US economy, implying $\eta = 0.38$ (Grossmann and Steger 2017). We choose γ to match the share of residential land value in total housing value $(1-\gamma)$. Using time series of the aggregate residential land value and the total value of housing from Davis and Heathcote (2007) reveals that the share of residential land in total housing value has been increasing from 10 percent in 1950 to around 30 percent in 1975. Since then it has been fluctuating between 25 and 40 percent. We target an average value of one third, implying a value of γ equal to 0.78.

We choose g^X such that we match (given γ, η, and g^Y) an annual average growth rate of rents of 1 percent. According to Knoll (2017), the annual average growth rate of real rents in the postwar (1953-2017) US economy was about 0.8 percent. Albouy, Ehrlich, and Liu (2016) argue that official data on rent growth is biased downwards due to incomplete accounting for quality improvements. Assuming that rents grow by one percent annually (given γ, η, and g^Y) and using (22) implies $g^X = 0.009$.[34]

The parameter ξ captures the importance of adjustment costs associated with land reallocations between the housing and the numeraire sector. This parameter is difficult to calibrate, as it does not affect the steady state and has an impact only along the transition. We calibrate ξ such that the (average) speed of convergence of residential land, N, computed in Section 6.2, is identical to the speed of convergence implied by the

[33] The data is obtained from FRED (https://fred.stlouisfed.org/), series *A93RX0Q048SBEA_P* (accessed on November 23, 2018).

[34] The low value for g^X in comparison to g^Y is supported by evidence of low, sometimes even negative, productivity growth in the construction sector (Davis and Heathcote 2005).

empirical data for the period 1945 to 1975. This yields $\xi = 765$.[35]

We calibrate κ to match the observed allocation of land in the residential sector. According to geographic land-use data provided by Falcone (2015), 30.2 percent of the total US surface is used economically and 16.9 percent of this land is used as residential land. Hence, we set $\kappa = 0.169$.[36]

6.2 Abolishing Zoning Regulations

Residential zoning regulations are widely considered as an important amplifier of surging housing costs in a growing economy (Glaeser, Gyourko, and Saks 2005; Saiz 2010). For instance, Albouy and Ehrlich (2018) find that, based on data for 230 metropolitan areas in the US from 2005 to 2010, observed land-use restrictions substantially increase housing costs. Moreover, Gyourko and Molloy (2015) argue that zoning regulations were effectively introduced in the US during the 1970s. This is consistent with the data provided by Davis and Heathcote (2007) showing that residential land grew by an average annual growth rate of 5 percent during time period 1945-1975 and grew merely by an average annual growth rate of 0.7 percent during time period 1976-2016.

To address the two research questions set up in the introduction, we compare two scenarios. In the baseline scenario (zoning), the economy is in a steady state, conditional on the binding zoning regulation $N = \kappa Z$ with $\kappa = 0.169$. In the policy-reform scenario (no zoning), residential zoning regulations are abolished completely. That is, we set $\kappa = 1$ such that the zoning constraint $N \leq \kappa Z$ is not binding anymore. The policy-reform scenario exhibits transitional dynamics, starting from the steady state of the baseline scenario. The analysis captures all general equilibrium effects. That is, all prices

[35] We assume that the long-run dynamics in N came to a halt by the introduction of zoning regulations in the 1970s (Gyourko and Molloy 2015). We then combine the steady state from the model with the observed data between 1945 to 1974 (Davis and Heathcote 2007) to determine the average speed of convergence. This shows that after 30 years about 31 percent of the gap between the initial N and the steady state has been closed. Hence, we set ξ such that residential land, N, has closed 31 percent of the gap between start value and steady state after 30 years of the transition in the experiment of Section 6.2.

[36] Without zoning regulations the model would imply a steady state share, N/Z, equal to 62 percent. Land use regulations started to play a major role in the residential sector in the 1970s (Gyourko and Molloy 2015). Consistent with this observation, average annual growth of residential land was about 3-5 percent during 1945 to 1975 and is almost zero since then (Davis and Heathcote 2007).

$\{w, r, p\}$ are fully endogenous and change in response to an exogenous policy trigger.[37]

Comovement of rents and wealth inequality Figure 1 (a) displays the time path of rents in the baseline scenario (zoning) and in the policy-reform scenario (no zoning). It can be seen that rents grow temporarily at a slower pace in the policy-reform scenario (solid curve) compared to the baseline scenario (dashed curve). This is intuitive as the economy extends the supply of housing along the extensive margin in response to the abolishment of zoning.[38] Figure 1 (b) shows that wealth inequality (measured by the top 10 percent wealth share) declines by about 0.7 percentage points (from 73.7 percent to 73 percent) over time. That is, rent growth and wealth inequality are positively associated in this general equilibrium experiment.

The observation that slower rent growth goes hand in hand with declining wealth inequality is in contrast to the rent channel in partial equilibrium (cf. Section 4.1.2). This seemingly contradiction can be easily explained. The policy experiment under study gives rise to a set of price changes, in addition to the change in rents. The most important one, when it comes to understanding the dynamics of wealth inequality, is the decline of future wages. The supply side deregulation under study triggers a housing boom in the sense that the production of aggregate housing services, $S = Nh = Nx^{\gamma} = X^{\gamma}N^{1-\gamma}$ (recall that $x = X/N$ denote structures per house), is being expanded. The resulting slower temporary rent growth implies that the competitive wage rate of construction workers, $w = q^X \frac{\partial I^X}{\partial L^X}$, declines relative to the baseline scenario. The reason is that the price of new residential buildings, measured by q^X, declines as well.[39] This process is reinforced by two additional mechanisms. First, there is a transformation of commercial land, Z^Y, into residential land, N, in the process of real estate development. Second, investments are temporarily channeled from the numeraire sector to real estate development.

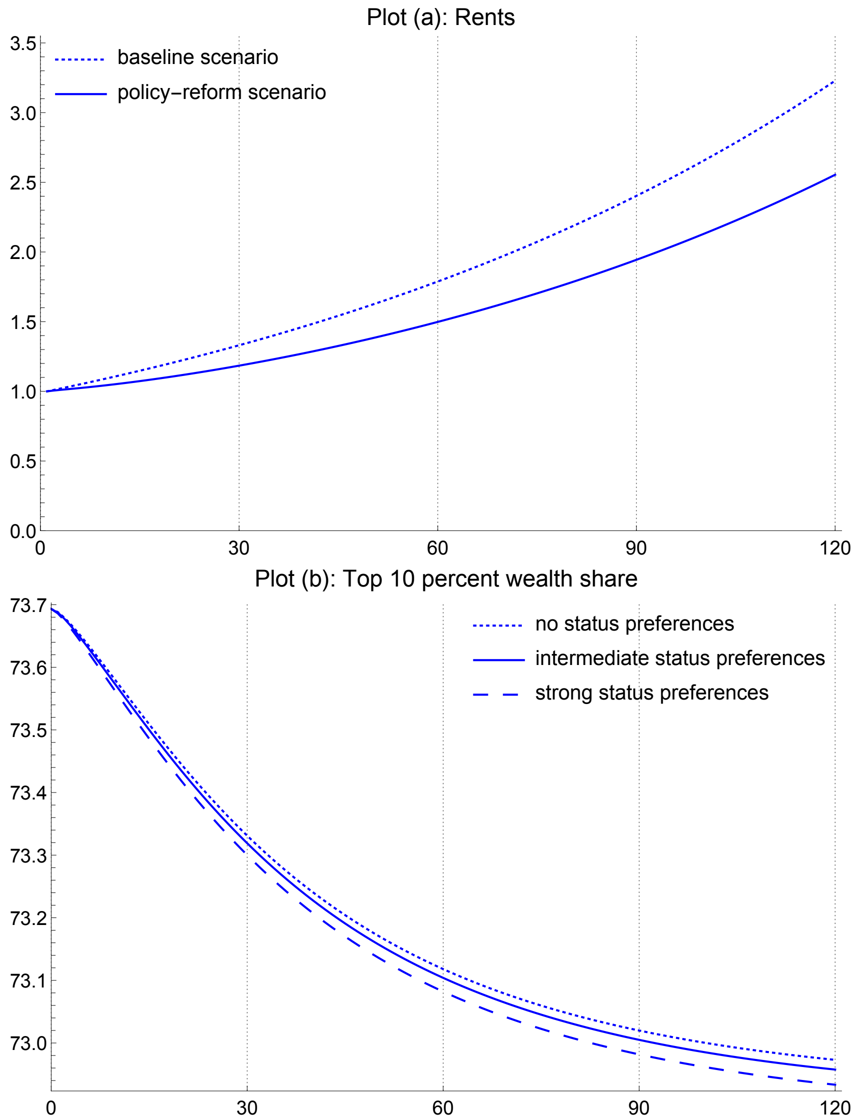
As both physical capital and land are complementary to labor in the numeraire sector,

[37] Favilukis, Mabille, and Van Nieuwerburgh (2018) consider a similar policy experiment in a model that is calibrated to New York. They model zoning regulations to exert an effect on labor productivity in the construction sector. In our model, relaxing zoning regulation is captured by an increase in κ, which constrains the amount of land allocated to the housing sector.

[38] Notice, however, that this is a temporary effect. The steady state growth rate of rents, given by (22), is unaffected.

[39] Recall $q^X(t) = \int_t^{\infty} R^X(\tau) e^{\int_t^{\tau} -(r(v)+\delta^X)\mathrm{d}v} \mathrm{d}\tau$, where $R^X = p\gamma x^{\gamma-1}$ is the rental price for structures.

Figure 1: Rent and wealth dynamics in response to abolishment of zoning regulation.



Notes. Panel (a): Evolution of the housing rent in the baseline scenario (zoning) and the policy-reform scenario (no zoning), holding the housing expenditure share of the representative consumer constant (at $\bar{e} = 0.19$) and recalibrating θ according to (23). Panel (b): Evolution of the top 10 percent wealth share in response to the abolishment of the zoning regulation for $\phi = 0$ (no status concerns), $\phi = 0.104$ (intermediate status concerns), $\phi = 0.26$ (strong status concerns), holding the housing expenditure share of the representative consumer constant (at $\bar{e} = 0.19$) and recalibrating θ according to (23). Calibration otherwise as described in Section 6.1.

the competitive wage of workers in the numeraire sector, $w = \frac{\partial Y}{\partial L^Y}$, declines as well. Lower future wages exert a convergence force as poorer households increase their saving rate by relatively more in order to smooth consumption over time. Hence, the divergence mechanism described in Section 4.1.1 (higher saving rates for the rich) is weakened. This effect dominates the reinforcement of the divergence mechanism due to lower rent growth.

To see that lower future wages act indeed as a convergence force, let us return to (12) and recall that $\frac{\partial G(\omega_j,\cdot)}{\partial \omega_j} = \frac{\mu\tilde{w}-w}{\omega_j^2}$ provides an overall measure for the change in the wealth distribution, as explained in Section 4.1.1. By noting the definition of $\tilde{w}$, given by (5), shows that $\frac{\partial^2 G(\omega_j(t),w(\tau),\cdot)}{\partial \omega_j(t)\partial w(\tau)} = \frac{\mu(t)}{[\omega_j(t)]^2}e^{-\bar{r}(\tau,t)} > 0$. That is, a decline of future wages, $w(\tau)$, contributes to less wealth inequality in period t.[40]

Notably, according to Figure 1 (b), status concerns play only a minor role for the effect of abolishing zoning regulations on wealth inequality. This is in stark contrast to welfare effects to which we turn next.

Welfare Figure 2 displays the welfare gain, expressed as consumption-equivalent variations, as a function of the relative initial wealth position, $W_j/\bar{W}$, from abolishing zoning regulations. We distinguish, first, between the partial equilibrium effects and the general equilibrium effects and, second, between the case of no status preferences ($\phi = 0$) and the case of an intermediate level of status preferences ($\phi = 0.104$).

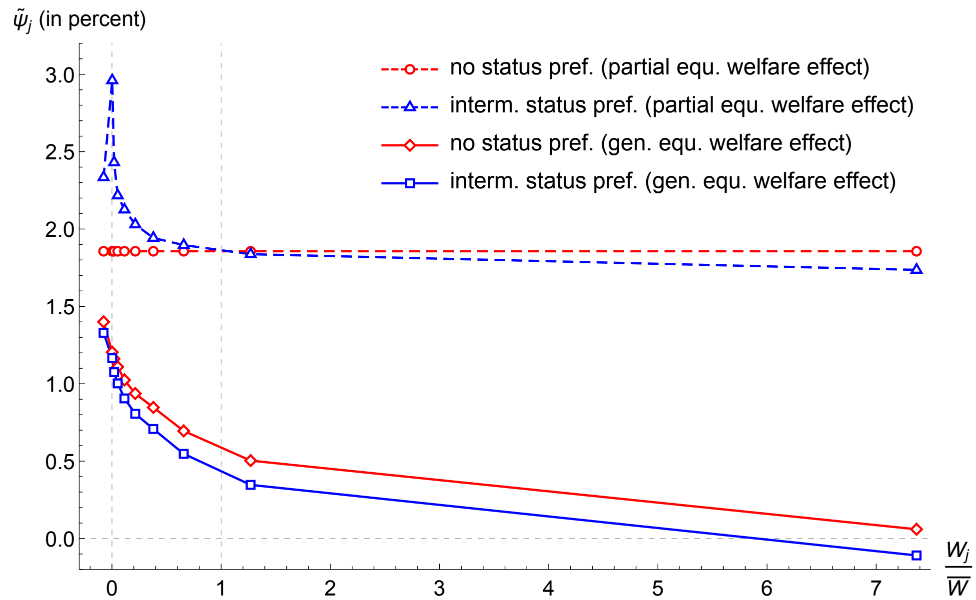
Consider first the general equilibrium welfare effects of the abolishment of zoning regulations under status preferences (see the bottom curve marked by squares). Welfare of the representative household (possessing average wealth, $W_j/\bar{W} = 1$) increases by almost 0.5 percent in general equilibrium under status concerns. However, the welfare effects are asymmetric. The poor benefit by more than the rich. The reason is twofold. The policy reform triggers slower rent growth. The resulting favorable price index effect is especially pronounced for the poor, due to their higher housing expenditure share.[41] The policy reform also reduces house prices. The decline in house prices exerts a negative wealth effect and this effect is stronger for the wealth rich.[42] Taken together, the price

[40]In contrast, the initial wage drop exerts a divergence force. Households smooth consumption by saving less today. This effect is stronger for the poorer households such that saving rates of poorer households decrease by relatively more, as their main income source is labor income. Given that the initial wage drop is small, this effect is weak.

[41]Recall, from Proposition 2, that $\frac{\partial \tilde{\mathcal{P}}(p,e)}{\partial e \partial p} > 0$.

[42]The policy reform triggers a drop in the house price, which reduces non-human wealth (W_j). It also triggers a fall in wages, which reduces human wealth ($l_j\tilde{w}$). The effect on non-human wealth is stronger. The calibration implies that the percentage of an agent's overall wealth in the form of (non-human) wealth increases with (non-human) wealth. Taken together, the policy reform under study lowers overall

Figure 2: Welfare gain, expressed as consumption-equivalent variations and denoted by $\tilde{\psi}_j$, as a function of the (non-human) wealth position, $W_j/\bar{W}$, from abolishing zoning regulations for $\phi = 0$ (no status concerns) and for $\phi = 0.104$ (intermediate status concerns).



Note: Measure $\tilde{\psi}_j$ is defined by $\int_t^\infty \frac{\left[(1+\tilde{\psi}_j)\mathcal{C}_j^0(\tau)\right]^{1-\sigma}-1}{1-\sigma} e^{-\rho(\tau-t)}\mathrm{d}\tau = \int_t^\infty \frac{\mathcal{C}_j^1(\tau)^{1-\sigma}-1}{1-\sigma} e^{-\rho(\tau-t)}\mathrm{d}\tau$, where $\mathcal{C}_j^i(\tau)$ denotes ideal consumption index of agent j at time τ in scenario $i \in \{0, 1\}$. The baseline scenario is indexed by superscript 0, the policy-reform scenario is indexed by superscript 1. The calibration is described in Section 6.1.

index effect and the wealth effect are both responsible for the asymmetry in welfare gains between wealth poor and wealth rich households. Notably, the overall welfare gain is even negative for the richest decile. That is, the wealth rich loose due to lower house prices despite lower rents, or more generally speaking, despite lower housing costs.

The isolated price index effect is shown by the curve marked by triangles (partial equilibrium). There we evaluate the welfare gain by accounting for a slower rent growth in the policy-reform scenario relative to the baseline scenario, as shown in Figure 1 (a), but keep everything else according the baseline scenario. We find that slower rent growth in response to abolishing zoning regulations, everything else the same, produces an average welfare gain of about 1.8 percent, i.e. a much higher welfare gain than in

wealth $(W_j + l_j\tilde{w})$ and this effect is stronger for the wealth rich.

general equilibrium.[43]

The picture changes if we set $\phi = 0$ (no status preferences). The price index effect is now symmetric across wealth groups (horizontal line marked by circles). This partial equilibrium exercise hence shows that Schwabe's law implies that lower rent growth benefits poor households more than rich households. Moreover, the general equilibrium welfare gain from the abolishment of zoning regulations is stronger without status preferences (as indicated by the curve marked by diamonds). In the case with status preferences, there is overconsumption of housing services, due to the negative externality associated with housing. From this perspective, a zoning constraint is a good thing, as it address an inefficiency as a second best instrument (Schünemann and Trimborn 2017).[44] Hence, removing the zoning regulation produces a smaller welfare gain under status preferences, as can be recognized by comparing to two lower curves in Figure 2.

7 Renters vs. Homeowners

Two thirds of US households are homeowners while only one third are renters.[45] In fact, our model can equivalently be interpreted as an economy of homeowners. All results still hold true, independently of whether we consider households as renters or homeowners.

Assume that all housing is owner-occupied such that $s_j = N_j h$, where N_j is the amount of housing owned by group j and h is the flow of housing services derived from one unit of housing.[46] Instead of choosing the flow of housing services (s_j) when being a renter, a homeowner chooses the stock of housing (N_j) that she owns. The household

[43] The wage grows at a slower pace and the interest rate is slightly higher along the transition in the policy-reform scenario. Both effects suppress welfare. Notice also that the welfare gain is falling from the second wealth decile onwards, but is higher for the second decile compared to the first decile. This non-monotonicity of the curve marked by triangles in Figure 2 is an implication of calibrating the joint distribution of wealth and labor endowment. In the second wealth decile labor income is lower compared to the first decile and falling rent is therefore particularly welfare-enhancing.

[44] The second-best optimal zoning constraint, assuming $\phi = 0.104$, amounts to $\kappa = 0.49$.

[45] US Bureau of the Census, Homeownership Rate for the United States, retrieved from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/RHORUSQ156N), series RHORUSQ156N (accessed December 21, 2018).

[46] The assumption that all housing is owner-occupied is common in the macro-housing literature (Iacoviello 2005).

problem for homeowners then modifies to

$$\max_{\{c_j(t),N_j(t)\}_{t=0}^{\infty}} \int_0^{\infty} u\left(c_j(t), N_j(t)h(t)\right) \mathrm{e}^{-\rho t}\mathrm{d}t \tag{24}$$

$$\text{s.t.}\quad \dot{W}_j(t) = r(t)A_j(t) - p^N(t)N_j(t) + w(t)l_j - c_j(t), \tag{25}$$

$$A_j(t) = W_j(t) - P^H(t)N_j(t), \tag{26}$$

where $p^N \equiv rP^H + \delta^X q^X x + q^X \dot{x} - \dot{P}^H$ denotes the user cost of housing. It consists of the sum of foregone interest payments (rP^H) and expenditures for maintenance and expansion ($\delta^X q^X x + q^X \dot{x}$) minus appreciation gains ($\dot{P}^H$). The optimality conditions of the above-stated homeowner problem are identical to those conditions that the renter, who maximize U_j s.t. (3), obtains.[47] Since the optimality conditions are identical, all results presented above hold true if one models households as homeowners instead of renters. The difference is merely in the interpretation. Instead of the rent it is the user cost of housing that affects the distribution of wealth and welfare. For example, the result on the partial equilibrium effect of rising rent on wealth inequality (Proposition 5) is equivalent to the effect of rising user cost of housing.

Under financial frictions additional mechanisms start playing a role. For instance, the rent and the user cost per unit of housing services may diverge implying that renters pay a higher price for housing services. Similarly, if houses pay a rate of return that differs from the rate of return paid by other assets, the portfolio structure plays a role for the wealth effects of surging house prices (Kuhn, Schularick, and Steins 2018).

8 Conclusion

This paper aims at better understanding the welfare implications of surging housing costs as well as the long-term comovement between housing costs and wealth inequality in a full-fledged dynamic general equilibrium model. The demand side features heterogeneity in housing expenditure shares, consistent with Schwabe's law. We have shown that such

[47] This can be seen by replacing s_j with $N_j h$ and p with $\frac{ucost}{h}$ in the first-order condition of renters (Appendix 9.3). The equality $p = \frac{ucost}{h}$ is implied by $P^H \equiv q^N + q^X x$ together with the capital market no arbitrage conditions (cf. Appendix 9.1, Definition of General Equilibrium, Condition 5).

heterogeneity is important for welfare effects of changes in the path of housing costs, but less so for wealth inequality. The supply side features a distinction between the extensive margin of the housing stock (the number of houses) and the intensive margin (the size of the average house). This model structure lends itself to investigating the consequences of removing those policies that regulate the use of land for residential purposes and, therefore, primarily constrain the extensive margin of the housing stock.

We have clarified, in a first step, the partial equilibrium analytics of surging housing costs on wealth inequality and welfare. In the empirical relevant case of an intertemporal elasticity of substitution below unity, forward-looking behavior implies that rising housing costs lead to a decline in wealth inequality. This, at the first glance surprising effect, reflects that the poor increase their saving rates more than the rich to cope with surging housing costs in the future. We have also shown that larger heterogeneity in housing expenditure shares is associated with larger heterogeneity of welfare. The general equilibrium analysis, the second step, has investigated numerically the comovement between housing costs and wealth inequality and welfare, as triggered by the abolishment of zoning regulations. This policy reform, despite suppressing rent growth, lowers wealth inequality. The reason, in a nutshell, is that the policy reform suppresses future wage growth, in addition to lowering rent growth, such that households increase their saving rates to smooth consumption over time. This effect is especially pronounced for the wealth poor. Abolishing zoning regulations also lowers the inequality of welfare. The calibrated model implies that most households gain from the policy reform. Only the richest households are worse off.

9 Appendix

9.1 Definition of General Equilibrium

Definition A.1 *A general equilibrium is a sequence of aggregate and group-specific quantities, a sequence of prices, and a sequence of operating profits of housing services producers*

$$\{Y(t), K(t), X(t), N(t), x(t), h(t), M(t), L^Y(t), L^X(t), L^N(t), Z^Y(t)\}_{t=0}^{\infty},$$

$$\{\{c_j(t), s_j(t), W_j(t), K_j(t), Z_j^Y(t), N_j(t)\}_{j=1}^{J}\}_{t=0}^{\infty},$$

$$\{p(t), P^Z(t), q^N(t), q^X(t), w(t), r(t), R^Z(t), R^X(t)\}_{t=0}^{\infty}, \text{ and } \{\pi_t\}_{t=0}^{\infty},$$

respectively, for initial distributions $\{K_j(0), Z_j^Y(0), N_j(0)\}_{j=1}^{J}$ *such that*

1. *individuals maximize lifetime utilities; and for all* t
2. *the representative firms in the sectors supplying structures* (X)*, the numeraire good* (Y)*, developed real estates* (N)*, and housing services* (h) *maximize the PDV of their respective infinite profit stream, taking prices as given;*
3. *labor markets clear:* $L^Y(t) + L^X(t) + L^N(t) = L$ *with* $L = \sum_j n_j l_j$*;*
4. *all asset markets clear:* $K(t) = \sum_j n_j K_j(t)$, $N(t) = \sum_j n_j N_j(t)$, $Z^Y(t) = \sum_j n_j Z_j^Y(t) = Z - N(t)$*;*
5. *perfect arbitrage across all assets classes holds:* $\frac{\dot{q}^N(t)}{q^N(t)} + \frac{\pi(t)}{q^N(t)} = \frac{\dot{q}^X(t)}{q^X(t)} + \frac{R^X(t)}{q^X(t)} - \delta^X = \frac{\dot{P}^Z(t)}{P^Z(t)} + \frac{R^Z(t)}{P^Z(t)} = r(t)$*;*
6. *the market for housing services clears:* $S(t) \equiv \sum_j n_j s_j(t) = N(t)h(t)$*;*
7. *the market for the numeraire good clears:* $Y(t) = C(t) + I^K(t) + M(t)$*, where* $C(t) \equiv \sum_j n_j c_j(t)$.[48]

9.2 Steady State

Proposition A.1 (Steady state) *Assume that productivity parameters* B^X *and* B^Y *grow at constant exponential growth rates* $g^X \geq 0$ *and* $g^Y \geq 0$*, respectively. The unique steady state growth rates then read as follows.*

(i) Variables $\{K, C, M, q^N, P^Z, R^Z, P^H, w, W\}$ *grow at the rate* g^Y*,*

(ii) Variables $\{X, x\}$ *grow at the rate* $\eta g^Y + (1-\eta) g^X$*,*

[48] The goods market clearing condition is redundant, according to Walras' law. To exclude conceptual or calculation errors, we analytically checked that the long run equilibrium derived from conditions 1-6 fulfills condition 7.

(iii) Variable $\{p\}$ *grows at the rate* $(1-\gamma\eta)\, g^{Y}-\gamma\,(1-\eta)\, g^{X}$,
(iv) Variables $\left\{q^{X}, R^{X}\right\}$ *grow at the rate* $(1-\eta)\left(g^{Y}-g^{X}\right)$,
(v) Variables $\{h, S\}$ *grow at the rate* $\gamma\left[\eta g^{Y}+(1-\eta)\, g^{X}\right]$,
(vi) Variables $\left\{N, Z^{Y}, L^{Y}, L^{X}, L^{N}, r\right\}$ *grow at zero rate.*

The proof can be found in Section 9.3, along with the other proofs of results in the main text. Notice also that Proposition A.1 implies that the steady state growth rate of GDP, $GDP=Y+pNh+wL^{X}$, equals g^{Y}.[49]

9.3 Proofs

Proof of Proposition 1 (Housing expenditure shares). The dynamic optimization problem of any agent j reads as follows

$$\max_{\{c_j(t),s_j(t)\}_{t=0}^{\infty}} \int_0^{\infty} \frac{[(c_j(t))^{1-\theta}(s_j(t)-\phi\bar{s}(t))^{\theta}]^{1-\sigma}-1}{1-\sigma}\mathrm{e}^{-\rho t}\mathrm{d}t \quad \text{s.t.} \tag{27}$$

$$\dot{W}_j(t) = r(t)W_j(t)+w(t)l_j-c_j(t)-p(t)s_j(t), \tag{28}$$

$\lim_{t\to\infty} W_j(t)\mathrm{e}^{-\int_0^t r(v)\mathrm{d}v} \geq 0$, $W_j(0)$ given. The associated current-value Hamiltonian reads as

$$\mathcal{H}_j = \frac{[(c_j)^{1-\theta}(s_j-\phi\bar{s})^{\theta}]^{1-\sigma}-1}{1-\sigma}+\lambda_j[rW_j+wl_j-c_j-ps_j]. \tag{29}$$

The first-order optimality conditions can be written as

$$\frac{\theta}{1-\theta}\frac{c_j}{s_j-\phi\bar{s}} = p \text{ , i.e., } c_j = p\frac{1-\theta}{\theta}(s_j-\phi\bar{s}), \tag{30}$$

$$(1-\theta)(c_j)^{-\theta-\sigma(1-\theta)}(s_j-\phi\bar{s})^{\theta(1-\sigma)} = \lambda_j, \tag{31}$$

$$-\frac{\dot{\lambda}_j}{\lambda_j} = r-\rho \tag{32}$$

$$\lim_{t\to\infty} W_j(t)\lambda_j(t)\mathrm{e}^{-\rho t} = 0. \tag{33}$$

(30) confirms that expansion paths are co-linear. Summing over all j, (30) implies that average consumption levels, $\bar{c}$ and $\bar{s}$, are related according to

$$\bar{c} = \frac{(1-\phi)(1-\theta)}{\theta}p\bar{s}. \tag{34}$$

Eq. (34) describes, for constant p, the expansion path of the representative consumer, who owns the average wealth and articulates the average demand (e.g. Mas-Colell, Whinston,

[49]The steady state growth rate of each GDP component (Y, pNh, wL^{X}) equals g^{Y}.

and Green 1995, Chapter 4). Using (30) in (31) we also obtain

$$\lambda_j = (1-\theta)^{1+(\sigma-1)\theta}\theta^{(1-\sigma)\theta}(c_j)^{-\sigma}p^{(\sigma-1)\theta}, \text{ i.e. } -\frac{\dot{\lambda}_j}{\lambda_j} = \sigma\frac{\dot{c}_j}{c_j} + (1-\sigma)\theta\frac{\dot{p}}{p}. \tag{35}$$

Combining (32) and (35) we have

$$\frac{\dot{c}_j}{c_j} = \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma}\frac{\dot{p}}{p} \equiv g^c. \tag{36}$$

Denote average housing expenditure by $E \equiv p\bar{s}$ and its growth rate by g^E. According to (34) and (36), we obtain

$$g^E = g^c = \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma}\frac{\dot{p}}{p}. \tag{37}$$

Now define $\mathfrak{s}_j \equiv s_j/\bar{s}$ (consumption of housing services of agent j relative to the average) and use (30) to write

$$e_j = \frac{ps_j}{c_j + ps_j} = \frac{1}{\frac{c_j}{ps_j}+1} = \frac{1}{\frac{1-\theta}{\theta}\frac{s_j-\phi\bar{s}}{s_j}+1} = \frac{\theta}{1-\frac{(1-\theta)\phi}{\mathfrak{s}_j}}. \tag{38}$$

According to (30), $E = p\bar{s}$ and $\mathfrak{s}_j = s_j/\bar{s}$, we also have

$$c_j = \frac{1-\theta}{\theta}(\mathfrak{s}_j - \phi)E, \text{ i.e.} \tag{39}$$

$$\log c_j = \log\left[\frac{1-\theta}{\theta}E\cdot(\mathfrak{s}_j-\phi)\right] = \log\frac{1-\theta}{\theta} + \log E + \log(\mathfrak{s}_j - \phi). \tag{40}$$

Taking the derivative with respect to time τ, we obtain

$$g^c = g^E + \frac{\mathrm{d}\log(\mathfrak{s}_j-\phi)}{\mathrm{d}\tau}\text{ , i.e., } \frac{\mathrm{d}\log(\mathfrak{s}_j-\phi)}{\mathrm{d}\tau} = 0, \tag{41}$$

according to (37). Thus, $\mathfrak{s}_j$ is time-invariant.

Next, define $\mathcal{C}_j \equiv (c_j)^{1-\theta}(s_j - \phi\bar{s})^\theta$ (inner instantaneous utility) and use (30) to obtain

$$\mathcal{C}_j = \theta^{\theta-1}(1-\theta)^{1-\theta}p^{1-\theta}\bar{s}(\mathfrak{s}_j-\phi). \tag{42}$$

Using $E = p\bar{s}$, we can rewrite (42) as

$$\mathcal{C}_j = \theta^{\theta-1}(1-\theta)^{1-\theta}Ep^{-\theta}(\mathfrak{s}_j-\phi). \tag{43}$$

Taking logs on both sides of (43) and the derivative with respect to time τ, we obtain

$$\frac{\dot{\mathcal{C}}_j}{\mathcal{C}_j} = g^E - \theta\frac{\dot{p}}{p} = \frac{r-\rho}{\sigma} - \frac{\theta}{\sigma}\frac{\dot{p}}{p}, \tag{44}$$

where we used (36). Also define consumption expenditure of agent j as $\mathcal{E}_j \equiv c_j + ps_j$ and $\mathcal{P}_j \equiv \mathcal{E}_j/\mathcal{C}_j$. According to (3), we can then write (with period index τ)

$$\dot{W}_j(\tau) = r(\tau)W_j(\tau) + w(\tau)l_j - \mathcal{P}_j(\tau)\mathcal{C}_j(\tau). \tag{45}$$

Multiplying both sides of (45) by $\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}$ and integrating from period t forward yields

$$\int_t^\infty \dot{W}_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau = \int_t^\infty r(\tau)W_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau + \int_t^\infty \left[w(\tau)l_j - \mathcal{P}_j(\tau)\mathcal{C}_j(\tau)\right]\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau, \tag{46}$$

Integrating by parts implies that

$$\begin{aligned}\int_t^\infty \dot{W}_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau &= \lim_{T\to\infty}\left[W_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\right]_t^T + \int_t^\infty r(\tau)W_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau \\ &= -W_j(t) + \int_t^\infty r(\tau)W_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau, \end{aligned} \tag{47}$$

where the latter equation uses the transversality condition $\lim_{T\to\infty} W_j(\tau)\mathrm{e}^{-\int_t^T r(v)\mathrm{d}v}\mathrm{d}\tau = 0$. Using (47), $\tilde{w}(t) = \int_t^\infty w(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau$ and $\mathcal{W}_j(t) = W_j(t) + \tilde{w}(t)l_j$ as defined in (5) and (4), respectively, in (46) implies

$$\int_t^\infty \mathcal{P}_j(\tau)\mathcal{C}_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau = \mathcal{W}_j(t). \tag{48}$$

The solution of differential equation (44) is

$$\mathcal{C}_j(\tau) = \mathcal{C}_j(t)\mathrm{e}^{\frac{1}{\sigma}\int_t^\tau\left[r(v)-\rho-\theta\frac{\dot{p}(v)}{p(v)}\right]\mathrm{d}v}. \tag{49}$$

Substituting (49) into (48) and multiplying both sides with $\mathcal{P}_j(t)$ gives us

$$[\mathcal{E}_j(t) =]\, \mathcal{P}_j(t)\mathcal{C}_j(t) = \frac{\mathcal{W}_j(t)}{\int_t^\infty \frac{\mathcal{P}_j(\tau)}{\mathcal{P}_j(t)}\mathrm{e}^{\frac{1}{\sigma}\int_t^\tau\left[(1-\sigma)r(v)-\rho-\theta\frac{\dot{p}(v)}{p(v)}\right]\mathrm{d}v}\mathrm{d}\tau}. \tag{50}$$

Next, using $\mathfrak{s}_j = s_j/\bar{s}$ in (30), group-specific consumption expenditures, $\mathcal{E}_j = c_j + ps_j$, read as

$$\mathcal{E}_j = \frac{p\bar{s}}{\theta}[\mathfrak{s}_j - (1-\theta)\phi]. \tag{51}$$

Using (42) and (51) in $\mathcal{P}_j = \mathcal{E}_j/\mathcal{C}_j$ implies

$$\mathcal{P}_j = \frac{p^\theta}{\theta^\theta(1-\theta)^{1-\theta}} \frac{\mathfrak{s}_j - (1-\theta)\phi}{\mathfrak{s}_j - \phi}. \tag{52}$$

According to (52) and the fact that $\mathfrak{s}_j$ is time-invariant, we have

$$\frac{\mathcal{P}_j(\tau)}{\mathcal{P}_j(t)} = \left(\frac{p(\tau)}{p(t)}\right)^\theta. \tag{53}$$

Using (53) in (50) we find that

$$\mathcal{E}_j(t) = \frac{\mathcal{W}_j(t)}{\int\limits_t^\infty \left(\frac{p(\tau)}{p(t)}\right)^\theta \mathrm{e}^{-\frac{1}{\sigma}\int_t^\tau \left[\rho + \theta\frac{\dot{p}(v)}{p(v)} + (\sigma-1)r(v)\right]\mathrm{d}v} \mathrm{d}\tau}. \tag{54}$$

According to (51), for two different agents, j and k, we have

$$\frac{\mathcal{E}_j}{\mathcal{E}_k}\left[= \frac{\mathcal{P}_j\mathcal{C}_j}{\mathcal{P}_k\mathcal{C}_k}\right] = \frac{\mathfrak{s}_j - \phi(1-\theta)}{\mathfrak{s}_k - \phi(1-\theta)}, \tag{55}$$

Using (54) and recalling that a household k with $\mathfrak{s}_k = 1$ has average total wealth $\bar{\mathcal{W}}(t)$, (55) can be written as

$$\frac{\mathfrak{s}_j - (1-\theta)\phi}{1-(1-\theta)\phi} = \frac{\mathcal{W}_j(t)}{\bar{\mathcal{W}}(t)} \equiv \Omega_j(t). \tag{56}$$

Since $\mathfrak{s}_j$ is time-invariant, according to (41), (56) implies that total wealth must grow at the same rate for all j. Using this fact in (56) and solving for $\mathfrak{s}_j$ implies

$$\mathfrak{s}_j = (1-\theta)\phi + [1-(1-\theta)\phi]\,\Omega_j(0). \tag{57}$$

Substituting (57) into (38) confirms (6). ■

Proof of Remark 1 (Representative Household). Defining using $C \equiv \sum_j n_j c_j$ and using $S \equiv \sum_j n_j s_j$, and summing the left and right hand sides of (28), (30) and (36) over all j yields:

$$\frac{C}{1-\theta} = \frac{p(1-\phi)}{\theta}S, \tag{58}$$

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma}\frac{\dot{p}}{p}, \tag{59}$$

$$\dot{W} = rW + wL - C - pS. \tag{60}$$

These FOC are identical to the FOC that result from the problem of a single household who owns the entire endowments ($\sum_j l_j$ and $\sum_j W_j$) and makes the aggregate consumption and saving decisions by taking the reference level of housing consumption, $\bar{s}(t)$, as exogenous. ■

Proof of Proposition 2 (Ideal price indices). We can rearrange (38) to obtain

$$\mathfrak{s}_j = \frac{(1-\theta)\phi}{1-\frac{\theta}{e_j}} \tag{61}$$

Substituting (61) into (52) confirms (7). ■

Proof of Proposition 3 (Propensity to consume). According to (54) and definition $\mu_j = \mathcal{E}_j/\mathcal{W}_j$, we find

$$\mu_j(t) = \frac{1}{\int\limits_t^\infty \left(\frac{p(\tau)}{p(t)}\right)^\theta \mathrm{e}^{-\frac{1}{\sigma}\int_t^\tau \left[\rho+\theta\frac{\dot{p}(v)}{p(v)}+(\sigma-1)r(v)\right]\mathrm{d}v}\mathrm{d}\tau}. \tag{62}$$

Using $\bar{r}(\tau,t) = \int_t^\tau r(v)d\tau$ and rearranging terms in (62) by using

$$\exp\left(-\frac{\theta}{\sigma}\int_t^\tau \left[\frac{\dot{p}(v)}{p(v)}\right]\mathrm{d}v\right) = \left(\frac{p(\tau)}{p(t)}\right)^{-\frac{\theta}{\sigma}} = \bar{p}(\tau,t)^{-\frac{\theta}{\sigma}} \tag{63}$$

confirms (8). ■

Proof of Proposition 4 (Saving rates). Plug $\mathcal{E}_j = \mu_j\mathcal{W}_j$, $\mathcal{W}_j = W_j + \tilde{w}l_j$, $y_j = rW_j + wl_j$, and $\omega_j = W_j/l_j$ into the definition of the saving rate, given by $sav_j = 1-\mathcal{E}_j/y_j$. ■

Proof of Proposition 5 (Rent channel). According to (8), the derivative of the propensity to consume with respect to $p(\tau)$ is given by

$$\frac{\partial\mu(t)}{\partial p(\tau)} = -\frac{(\sigma-1)\theta}{\sigma\mu(t)^2}\int\limits_t^\infty \frac{\left(\frac{p(t)}{p(\tau)}\right)^{\frac{\theta(1-\sigma)}{\sigma}}}{p(\tau)\exp\left[\frac{\sigma-1}{\sigma}\bar{r}(\tau,t)+\frac{\rho}{\sigma}(\tau-t)\right]^{\frac{\sigma-1}{\sigma}}}\mathrm{d}\tau \tag{64}$$

implying that

$$\frac{\partial \mu(t)}{\partial p(\tau)} \begin{cases} < 0 & \text{for} \quad \sigma > 1 \\ = 0 & \text{for} \quad \sigma = 1 \\ > 0 & \text{for} \quad \sigma < 1 \end{cases} . \tag{65}$$

From (12) one gets

$$\frac{\partial^2 G(\omega_j(t), \cdot)}{\partial \omega_j(t) \partial p(\tau)} = \frac{\tilde{w}(t)}{\omega_j(t)^2} \frac{\partial \mu(t)}{\partial p(\tau)} \begin{cases} < 0 & \text{for} \quad \sigma > 1 \\ = 0 & \text{for} \quad \sigma = 1 \\ > 0 & \text{for} \quad \sigma < 1 \end{cases} . \tag{66}$$

This concludes the proof. ■

Proof of Proposition 6 (Welfare). Recall that $\mathcal{C}_j(\tau)$ denotes the ideal consumption index of agent j at time τ and lifetime utility of agent j is

$$U_j(t) \equiv \int_t^\infty \frac{\mathcal{C}_j(\tau)^{1-\sigma} - 1}{1-\sigma} \mathrm{e}^{-\rho(\tau - t)} \mathrm{d}\tau. \tag{67}$$

Using $\bar{r}(\tau, t) = \int_t^\tau r(v) dv$ and (63), (49) can be written as

$$\mathcal{C}_j(\tau) = \mathcal{C}_j(t) \bar{p}(\tau, t)^{-\frac{\theta}{\sigma}} \mathrm{e}^{\frac{\bar{r}(\tau,t) - \rho(\tau - t)}{\sigma}}. \tag{68}$$

Substituting (68) into (67) and using (13), we find $1 + \psi_j(t) = \mathcal{C}_j(t)/\bar{\mathcal{C}}(t)$. Using $\mathcal{E}_j = \mathcal{P}_j \mathcal{C}_j = \mu \mathcal{W}_j$, according to the definition of $\mu_j = \mathcal{E}_j/\mathcal{W}_j$ and the result $\mu_j = \mu$ for all j (Proposition 3) confirms (14). ■

Proof of Proposition A.1 (Steady state). Let a "hat" above a variable denote the steady state growth rate of this variable. According to (36), we obtain steady state interest rate

$$r = \rho + \theta(1 - \sigma)\hat{p} + \sigma \hat{C} \equiv r^* \tag{69}$$

(i.e. $\hat{r} = 0$). From profit maximization in the numeraire sector, the first-order conditions read as

$$r + \delta^K = \alpha \frac{Y}{K}, \ w = \beta \frac{Y}{L^Y}, \ R^Z = (1 - \alpha - \beta) \frac{Y}{Z^Y}, \tag{70}$$

implying

$$\hat{Y} = \hat{K} = \hat{w} = \hat{R^Z}. \tag{71}$$

Writing the production function $Y = K^\alpha \left(B^Y L^Y\right)^\beta \left(B^Y Z^Y\right)^{1-\alpha-\beta}$ in growth rates implies $\hat{Y} = \alpha \hat{K} + \beta(g^Y + \hat{L}^Y) + (1 - \alpha - \beta)(g^Y + \hat{Z}^Y)$. Supposing that the long run allocation of both labor and land is time invariant (which will be confirmed to be consistent with the derived steady state), i.e. $\hat{L}^Y = \hat{Z}^Y = 0$, and using $\hat{Y} = \hat{K}$ thus implies

$$\hat{Y} = g^Y. \tag{72}$$

Consider next the profit maximization problem of the representative real estate development firm in any period. Taking the price of developed real estates, q^N, and the land price, P^Z, as given, it chooses the number of new houses, $\dot{N}$, to maximize $q^N \dot{N} - \mathfrak{C}(\dot{N}, P^Z, w)$. Using (18), it solves

$$\max_{\dot{N}} \left\{ q^N \dot{N} - P^Z \dot{N} - w \frac{\xi}{2} (\dot{N})^2 \right\}. \tag{73}$$

The associated first-order condition implies the following law of motion (with N_0 given) for the number of developed real estates:

$$\dot{N} = \frac{q^N - P^Z}{w\xi}. \tag{74}$$

Since $\dot{N} = 0$ in steady state (time-invariant land allocation), we obtain $q^N = P^Z$ and

$$\hat{q}^N = \hat{P}^Z. \tag{75}$$

As the steady state interest rate is time-invariant, the asset market no arbitrage conditions $\dot{q}^N / q^N + \pi / q^N = \dot{P}^Z / P^Z + R^Z / P^Z = r$ in Definition A.1 imply that

$$\hat{P}^Z = \hat{R}^Z, \, \hat{\pi} = \hat{q}^N. \tag{76}$$

Summarizing (71), (72), (75) and (76), we obtain

$$\hat{P}^Z = \hat{R}^Z = \hat{R}^X = \hat{q}^X = \hat{\pi} = \hat{q}^N = \hat{K} = \hat{w} = \hat{R^Z} = g^Y. \tag{77}$$

The profit maximization problem in the construction sector is given by

$$\max_{\{M_s, L_s^X\}_{\tau=t}^{\infty}} \int_t^{\infty} \left(R_\tau^X X_\tau - M_\tau - w_\tau L_\tau^X \right) e^{\int_t^\tau -r_v \mathrm{d}v} \mathrm{d}\tau \quad \text{s.t.} \quad (20) \tag{78}$$

(with X_0 is given). The current-value Hamiltonian of the representative construction firm associated with optimization problem (78) together with the necessary first-order conditions can then be expressed as

$$\mathcal{H}^X \equiv R^X X - M - w L^X + q^X \left[M^\eta \left(B^X L^X \right)^{1-\eta} - \delta^X X \right], \tag{79}$$

$$\left[\frac{\partial \mathcal{H}^X}{\partial M}=\right] - 1 + \eta q^X \left(\frac{B^X L^X}{M}\right)^{1-\eta} = 0, \tag{80}$$

$$\left[\frac{\partial \mathcal{H}^X}{\partial L^X}=\right] - w + (1-\eta) q^X \left(B^X\right)^{1-\eta} \left(\frac{M}{L^X}\right)^{\eta} = 0, \tag{81}$$

$$\left[-\frac{\partial \mathcal{H}^X}{\partial X}=\right] - R^X + \delta^X q^X = \dot{q}^X - r q^X. \tag{82}$$

Differentiating (80) and (81) with respect to time,

$$\hat{q}^X = (\eta - 1)(g^X - \hat{M}) = \hat{w} - \eta \hat{M} + (\eta - 1) g^X. \tag{83}$$

Using (83) and recalling $\hat{w} = g^Y$ from (77), we obtain

$$\hat{M} = \hat{w} = g^Y, \tag{84}$$

$$\hat{q}^X = (\eta - 1)(g^X - g^Y). \tag{85}$$

Note that (82) implies $q^X(t) = \int_t^\infty R^X(\tau) \mathrm{e}^{\int_t^\tau -(r(v)+\delta^X)\mathrm{d}v} \mathrm{d}\tau$ in the absence of bubbles, as claimed in Section 5.2, leading to $\dot{q}^X/q^X + R^X/q^X - \delta^X = r$ (Definition A.1). Using (85), in steady state, we thus have $R^X/q^X = r + \delta^X - (\eta - 1)(g^X - g^Y)$ and

$$\hat{R}^X = \hat{q}^X = (\eta - 1)(g^X - g^Y). \tag{86}$$

Next, rewrite (20) to

$$\frac{\dot{X}}{X} = \frac{M^\eta (B^X L^X)^{1-\eta}}{X} - \delta^X. \tag{87}$$

In a steady state, thus, $\hat{X} = \eta \hat{M} + (1-\eta) g^X$. Hence, according to (84)

$$\hat{X} = \eta g^Y + (1-\eta) g^X = \hat{x}, \tag{88}$$

where $\hat{x} = \hat{X}$ follows from $x = X/N$ (structures per house) and a stationary allocation of land in the long run ($\hat{N} = 0$).[50] From the production and total demand of housing services $h = x^\gamma$ and $S = Nh$, we also obtain

$$\hat{h} = \hat{S} = \gamma \hat{x} = \gamma \left[\eta g^Y + (1-\eta) g^X\right]. \tag{89}$$

Recall that the profit per housing services producer is $\pi = (1-\gamma) p h$ (Section 5.1). Differentiating with respect to time, $\hat{p} = \hat{\pi} - \hat{h}$. Using (89) and $\hat{\pi} = g^Y$, according to (77), we confirm (22):

$$\hat{p} = (1 - \gamma\eta)\, g^Y - \gamma\,(1-\eta)\, g^X. \tag{90}$$

[50] This is also true in an economy with binding zoning regulations, where $N = \kappa Z$, with $\kappa \in (0, 1)$.

Differentiating (58) with respect to time and substituting (89) and (90), we find

$$\hat{C} = \hat{p} + \hat{S} = g^Y. \tag{91}$$

From (85) and (88), we have $\hat{q}^X + \hat{x} = g^Y$. Moreover, recall from (77) that $\hat{q}^N = g^Y$. Hence, we confirm that the house price, $P^H = q^N + q^X x$, grows in steady state at rate $\hat{P}^H = g^Y$. Finally, according to (60),

$$\frac{\dot{W}}{W} = r + \frac{wL}{W} - \frac{C}{W} - \frac{pS}{W}. \tag{92}$$

In view of (91), (92) requires that $\hat{W} = \hat{w}$. Recalling $\hat{w} = g^Y$ from (77), thus, $\hat{W} = g^Y$.■

Proof of Proposition 7. We need to verify that $\mu\tilde{w} = w$ holds in any steady state.

Notice first that the transversality condition of the household optimization problem requires $r^* - g^Y$. Substituting (90) and (91) into (69), we find that

$$r^* - g^Y = \rho - \left[(1 - \theta + \theta\gamma\eta)g^Y + \theta\gamma\,(1 - \eta)\,g^X\right](1 - \sigma) > 0 \tag{93}$$

always holds if $\sigma \geq 1$. Recall from Proposition A.1 that $\hat{w} = g^Y$. Thus, in a steady state, $w(\tau) = w(t)\mathrm{e}^{(\tau - t)g^Y}$ and, consequently, the PDV of wages, $\tilde{w}(t) \equiv \int_t^\infty w(\tau)\mathrm{e}^{\int_t^\tau -r(v)\mathrm{d}v}\mathrm{d}\tau$, can be written as

$$\tilde{w}(t) = \int_t^\infty \underbrace{w(t)\mathrm{e}^{g^Y(\tau - t)}}_{=w(\tau)}\mathrm{e}^{-r^*(\tau - t)}\mathrm{d}\tau = \frac{w(t)}{r^* - g^Y} \equiv \tilde{w}^*(t). \tag{94}$$

Also recall from (59) that, in steady state,

$$\hat{C} = \frac{r^* - \rho + (\sigma - 1)\theta\hat{p}}{\sigma} = g^Y, \tag{95}$$

where the latter follows from Proposition A.1.

Also note from (95) that $\left(\theta\hat{p} - r^* - \frac{\rho}{\sigma - 1}\right)\frac{\sigma - 1}{\sigma} = -(r^* - g^Y) < 0$ and consider next the propensity to consume, as given by (8). Using that, in steady state, $\bar{p}(\tau, t) = p(\tau)/p(t) = \mathrm{e}^{\hat{p}(\tau - t)}$ and $\bar{r}(\tau, t) \equiv \int_t^\tau r(v)dv = (\tau - t)r^*$, we can rewrite $\mu(t)$ in steady state as

$$\mu(t) = \left(\int\limits_t^\infty \mathrm{e}^{\left(\theta\hat{p} - r^* - \frac{\rho}{\sigma - 1}\right)\frac{\sigma - 1}{\sigma}(\tau - t)}\mathrm{d}\tau\right)^{-1} = r^* - g^Y \equiv \mu^*(t). \tag{96}$$

Using (94) and (96), we confirm $\mu^*(t)\tilde{w}^*(t) = w(t)$. ■

9.4 Calibration

Parameter	Value	Explanation/Target
L	1	Normalization
J	10	Match deciles
$\left\{W_j(0)/\bar{W}(0)\right\}_{j=1}^{J}$	see text	Wealth deciles, US, 2013 SCF
$\left\{l_j(0)/\bar{l}(0)\right\}_{j=1}^{J}$	see text	average earnings within wealth percentile, US, 2013 SCF
σ	2	$IES = 0.5$ (Havránek 2015)
Z	1	Normalization
δ^K	0.056	Davis and Heathcote (2005)
α	0.28	Land income share in Y sector (Grossmann and Steger 2017)
β	0.69	Labor expenditure share in Y sector (Grossmann and Steger 2017)
g^Y	0.02	Growth rate of GDP per capita (FRED)
δ^X	0.015	Hornstein (2009)
η	0.38	Labor expenditure share in X sector (Grossmann and Steger 2017)
g^X	0.009	Implied
κ	0.169	Share of residential land: 16.9 percent (Falcone 2015)
θ	$\{0.19, 0.17, 0.15\}$	Average housing expenditure share 0.19 (U.S. Bureau of Labor Statistics 2016)
ϕ	$\{0.000, \mathbf{0.104}, 0.260\}$	Difference between bottom and top income quintiles of Table 1
ρ	0.019	Real interest rate: 0.0577 (Jordà et al. 2018)
γ	0.78	Land's share in housing wealth: 1/3
ξ	765	Transition speed in N: 31 percent in 30 years (Davis and Heathcote 2007)

Table A.1: Set of parameters for the calibrated model.

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10 Online Appendix

10.1 Dynamic System

The macroeconomic model with housing is fully described by seven differential equations plus a set of static equations.[51]

$$\dot{X} = M^{\eta}(B^X L^X)^{1-\eta} - \delta^X X \tag{97}$$

$$\dot{N} = \frac{q^N - P^Z}{w\xi} \tag{98}$$

$$\dot{W} = rW + wL - C - pS \tag{99}$$

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma}\frac{\dot{p}}{p} \tag{100}$$

$$\dot{q}^X = -R^X + (r+\delta^X)q^X \tag{101}$$

$$\dot{q}^N = -\pi + rq^N \tag{102}$$

$$\dot{P}^Z = -R^Z + rP^Z \tag{103}$$

$$K = W - \left(q^N N + q^X X + P^Z Z^Y\right) \tag{104}$$

$$p = \frac{\theta}{(1-\theta)(1-\phi)}\frac{C}{S} \tag{105}$$

$$r + \delta^K = \alpha\frac{Y}{K} \tag{106}$$

$$w = \beta\frac{Y}{L^Y} \tag{107}$$

$$R^Z = (1-\alpha-\beta)\,\frac{Y}{Z^Y} \tag{108}$$

$$R^X = \gamma p x^{\gamma-1} \tag{109}$$

$$\pi = (1-\gamma)\,ph^{\gamma} \tag{110}$$

$$S = X^{\gamma}N^{1-\gamma} \tag{111}$$

$$L^X = \left(\frac{(1-\eta)q^X}{w}\right)^{\frac{1}{\eta}}\frac{M}{(B^X)^{\frac{\eta-1}{\eta}}} \tag{112}$$

$$M = \left(\eta q^X\right)^{\frac{1}{1-\eta}} B^X L^X \tag{113}$$

$$L = L^Y + L^X + L^N \tag{114}$$

[51] The dynamic system is derived in Grossmann and Steger (2017) for the case $\sigma = 1$ and $\phi = 0$ and can be readily extended to allow for $\sigma \neq 1$ and $\phi > 0$. In contrast to the aforementioned paper, we abstract from capital income taxation and normalize the land requirement per house to unity, $\psi = 1$ according to the notation in Grossmann and Steger (2017).

$$Z = Z^Y + N \tag{115}$$

$$x = \frac{X}{N} \tag{116}$$

$$h = x^\gamma \tag{117}$$

$$L^N = w\frac{\xi}{2}\left(\dot{N}\right)^2 \tag{118}$$

where $K(0)$, $N(0)$, $X(0)$ are given and Y is defined by (17).[52]

10.2 Computation of transitional dynamics for all agents

The computation of time paths for all J type of agents takes series for (normalized) prices and aggregate quantities – obtained from the solution of the representative agent economy in the first step – as given and derives time paths for each agent $j \in \{1, 2, ..., J\}$ by exploiting the recursive structure of the household problem. It is not necessary to employ numerical techniques like solving non-linear equation systems, interpolation, or numerical integration. Given the minor approximation error in the solution of the representative agent economy, the computed time paths for all J type of agents are hence exact to machine precision.

10.2.1 Discretization

In order to solve the model numerically we have to discretize the differential equation system that describes the economy. For a differential equation $\dot{x}(t) = f(x(t), y(t))$ we discretize according to $x_{t+1} - x_t = f(x(t), y(t))$.[53] The growth-adjusted first-order conditions read

$$\frac{\tilde{c}_j}{1-\theta} = \frac{\tilde{p}}{\theta}(\tilde{s}_j - \phi\tilde{\bar{s}}) \tag{119}$$

$$\frac{\dot{\tilde{c}}_j}{\tilde{c}_j} = \frac{r-\rho}{\sigma} + \frac{\sigma-1}{\sigma}\theta\left(\frac{\dot{\tilde{p}}}{\tilde{p}} + g^p\right) - g^c \tag{120}$$

$$\dot{\tilde{W}} = (r - g^c)\tilde{W}_j + \tilde{w}l_j - \tilde{c}_j - \tilde{p}\tilde{s}_j \tag{121}$$

$$0 = \lim_{t\to\infty} e^{-\tilde{\rho}t}\tilde{W}_{jt}\tilde{p}_t^{\theta(\sigma-1)}\left(\tilde{c}_{jt}\right)^{-\sigma} \tag{122}$$

$$\tilde{W}_{j0} = \text{given}, \tag{123}$$

where g^c is the exogenous growth rate of consumption (the numeraire) in the steady state, g^p is the steady state growth rate of rents, and $\tilde{\rho} \equiv \rho + (\sigma-1)(g^c - \theta g^p)$. The

[52] In total, there are 22 equations and 22 endogenous variables: X, N, W, C, q^X, q^N, P^Z, K, p, r, w, R^Z, R^X, π, S, L^X, L^N,M, L^Y, Z^Y, x, and h.

[53] We explored also different approximations, e.g. $x_{t+1} - x_t = f(\frac{x_{t+1}-x_t}{x_t}, \frac{y_{t+1}-y_t}{y_t})$ and the differences in the results are negligible.

discretized version hence reads (the ~ above variables is suppressed)

$$\frac{c_{jt}}{1-\theta} = \frac{p_t(s_{jt} - \phi \bar{s}_t)}{\theta} \tag{124}$$

$$c_{jt+1} - c_{jt} = \frac{r_t - \rho}{\sigma} c_{jt} - \frac{\theta(1-\sigma)}{\sigma} \left(\frac{p_{t+1} - p_t}{p_t} + g^p \right) c_{jt} - g^c c_{jt} \tag{125}$$

$$W_{jt+1} - W_{jt} = (r_t - g^c) W_{jt} + w_t l_j - c_{jt} - p_t s_{jt} \tag{126}$$

$$0 = \lim_{t \to \infty} e^{-\tilde{\rho} t} \tilde{W}_{jt} \tilde{p}_t^{\theta(\sigma-1)} \left(\tilde{c}_{jt} \right)^{-\sigma} \tag{127}$$

$$\tilde{W}_{j0} = given, \tag{128}$$

where the subscript "jt" now denotes the group j and (discrete) time t. This constitutes a linear, non-homogeneous system of first-order difference equations with time-variant coefficients and two boundary conditions. Rearranging yields

$$c_{jt+1} = \underbrace{\frac{(1-g^c)\sigma + r_t - \rho + \theta(\sigma - 1)\left(\frac{p_{t+1} - p_t}{p_t} + g^p \right)}{\sigma}}_{\equiv f_t} c_{jt} \tag{129}$$

$$W_{jt+1} = \underbrace{(1 + r_t - g^c)}_{\equiv g_t} W_{jt} - \frac{1}{1-\theta} c_{jt} + \underbrace{[w_t l_j - \phi p_t \bar{s}_t]}_{\equiv l_t^j} \tag{130}$$

$$0 = \lim_{t \to \infty} e^{-\rho t} W_{jt} p_t^{\theta(\sigma-1)} \left(c_{jt} \right)^{-\sigma} \tag{131}$$

$$W_{j0} = given. \tag{132}$$

The solution is

$$c_{jt} = c_{j0} \prod_{s=0}^{t-1} f_s \tag{133}$$

$$W_{jt} = W_{j0} \prod_{s=0}^{t-1} g_s - \frac{1}{1-\theta} \sum_{k=0}^{t-1} c_{jk} \prod_{s=k+1}^{t-1} g_s + \sum_{k=0}^{t-1} l_k^j \prod_{s=k+1}^{t-1} g_s. \tag{134}$$

10.2.2 Initial consumption

One obtains c_{j0} by applying the transversality condition (TVC) to (134) and plugging the solution for c_{jt} – as given by (133) – into the result. Define $\Theta_t^1 \equiv W_{j0} \prod_{s=0}^{t-1} g_s$, $\Theta_t^2 \equiv h \sum_{k=0}^{t-1} c_{jk} \prod_{s=k+1}^{t-1} g_s$ and $\Theta_t^3 \equiv \sum_{k=0}^{t-1} l_k^j \prod_{s=k+1}^{t-1} g_s$ and write (134) as

$$W_{jt} = \Theta_t^1 - \Theta_t^2 + \Theta_t^3. \tag{135}$$

We know that $\lim_{t\to\infty} W_{jt}\Xi_t = 0$, where $\Xi_t \equiv e^{-\rho t} p_t^{\theta(\sigma-1)} (c_{jt})^{-\sigma}$, such that

$$\lim_{t\to\infty} W_t^j \Xi_t = 0 = \lim_{t\to\infty} \Xi_t(\Theta_t^1 + \Theta_t^3) - \lim_{t\to\infty} \Xi_t \Theta_t^2 \tag{136}$$

$$\Leftrightarrow 1 = \frac{\lim_{t\to\infty} \Xi_t(\Theta_t^1 + \Theta_t^3)}{\lim_{t\to\infty} \Xi_t \Theta_t^2} \tag{137}$$

$$= \lim_{t\to\infty} \frac{\Xi_t(\Theta_t^1 + \Theta_t^3)}{\Xi_t \Theta_t^2} \tag{138}$$

$$= \lim_{t\to\infty} \frac{\Theta_t^1 + \Theta_t^3}{\Theta_t^2}. \tag{139}$$

Replacing Θ_t^1, Θ_t^2, and Θ_t^3 by their respective expressions yields

$$1 = \frac{W_{j0} \prod_{s=0}^{\infty} g_s + \sum_{k=0}^{\infty} l_k^j \prod_{s=k+1}^{\infty} g_s}{h \sum_{k=0}^{\infty} c_{jk} \prod_{s=k+1}^{\infty} g_s} = \frac{W_{j0} + \sum_{k=0}^{\infty} l_k^j \prod_{s=0}^{k} (g_s)^{-1}}{h \sum_{k=0}^{\infty} c_{jk} \prod_{s=0}^{k} (g_s)^{-1}}. \tag{140}$$

Inserting the solution for c_{jk} as given by (133) gives

$$c_{j0} = \frac{W_{j0} + \sum_{k=0}^{\infty} l_k^j \prod_{s=0}^{k} (g_s)^{-1}}{\sum_{k=0}^{\infty} \frac{1}{1-\theta} \frac{1}{f_k} \prod_{s=0}^{k} \frac{f_s}{g_s}} \tag{141}$$

10.2.3 How to deal with infinity

In the computation we have to assume that the dynamic system is in its steady state after a some period T, where the number of transition periods, T, is chosen sufficiently large. Then, sums and products can be modified to $\sum_{s=0}^{\infty} x_t = \sum_{s=0}^{T-1} x_t + \sum_{s=T}^{\infty} x$ and $\prod_{s=0}^{\infty} x_t = (\lim_{t\to\infty} x^t) \prod_{s=1}^{T-1} x_t$, where x denotes the respective steady state of x_t. The steady states for the time-dependent parameters are

$$f = 1 \tag{142}$$

$$g = 1 + r - g_c \tag{143}$$

$$l^j = wl^j - \phi p \bar{s}. \tag{144}$$

Accordingly, the denominator of (141) becomes

$$\sum_{k=0}^{\infty} \frac{h}{f_k} \prod_{s=0}^{k} \frac{f_s}{g_s} = \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^{k} \frac{f_s}{g_s} + \frac{1}{1-\theta} \sum_{k=T}^{\infty} f^{-1} \prod_{s=0}^{T-1} \frac{f_s}{g_s} \prod_{s=T}^{k} \frac{f}{g} \tag{145}$$

$$= \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^{k} \frac{f_s}{g_s} + \frac{1}{1-\theta} \left(\prod_{s=0}^{T-1} \frac{f_s}{g_s} \right) \sum_{k=T}^{\infty} [1 + r - g_c]^{T-k-1} \tag{146}$$

$$= \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^{k} \frac{f_s}{g_s} + \frac{1}{(1-\theta)(r-g_c)} \prod_{s=0}^{T-1} \frac{f_s}{g_s}. \tag{147}$$

Similarly, the second term in the numerator of (141) can be written as

$$\sum_{k=1}^{\infty} l_k^j \prod_{s=1}^{k} g_s^{-1} = \sum_{k=0}^{T-1} l_k^j \prod_{s=0}^{k} g_s^{-1} + \frac{l^j}{r-g_c} \prod_{s=0}^{T-1} g_s^{-1}.$$

Putting all together gives

$$c_{j0} = \frac{W_{j0} + \sum_{k=0}^{T-1} l_k^j \prod_{s=0}^{k} g_s^{-1} + \frac{l^j}{r-g_c} \prod_{s=0}^{T-1} g_s^{-1}}{\frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^{k} \frac{f_s}{g_s} + \frac{1}{(1-\theta)(r-g_c)} \prod_{s=0}^{T-1} \frac{f_s}{g_s}}. \tag{148}$$

10.2.4 Solution algorithm

For each $j \in \{1, 2, ..., J\}$:

1. Obtain initial consumption c_{j0} with (148).
2. Obtain individual consumption levels $\{c_{jt}\}_{t=0}^{T}$ from (133) or by iterating over the discretized Euler equation.
3. Obtain individual wealth levels $\{W_{jt}\}_{t=0}^{T}$ by making use of (134) or by iterating over the discretized budget constraint.
4. Obtain individual housing consumption $\{s_{jt}\}_{t=0}^{T}$ from the intra-temporal optimality condition.

10.3 Robustness

10.3.1 Status Preferences for Both Goods

If we replaced instantaneous utility (2) by

$$u(c_j, s_j) = \frac{[(c_j - \phi_c \bar{c})^{1-\theta} (s_j - \phi_s \bar{s})^{\theta}]^{1-\sigma} - 1}{1-\sigma}, \tag{149}$$

with $\phi_c, \phi_s \geq 0$, where $\bar{c}$ is average consumption of the numeraire good, then the housing expenditure share would still read as (6), with $\phi \equiv \frac{\phi_s - \phi_c}{1-\phi_c}$. Since $\phi > 0$ iff $\phi_s > \phi_c$, assuming status concerns with respect to housing services only ($\phi_c = 0$) captures, without loss of generality, that status concerns are higher for housing than for non-housing consumption.

10.3.2 CES Utility

Consider the following utility specification

$$u(c_j, s_j) = \frac{(\mathcal{C}_j)^{1-\sigma} - 1}{1-\sigma} \qquad \text{with} \qquad \mathcal{C}_j = \left[\theta (s_j - \phi \bar{s})^{1-\frac{1}{\kappa}} + (1-\theta) c_j^{1-\frac{1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}},$$

where $\kappa > 0$. The housing expenditure share of agent j (e_j) and the aggregate housing expenditure share ($\bar{e}$) are then given by

$$e_j = \frac{\theta^\kappa p^{1-\kappa}}{\theta^\kappa p^{1-\kappa} + (1-\theta)^\kappa \left(1 - \frac{\phi}{\mathfrak{s}_j}\right)}$$

$$\bar{e} = \frac{\theta^\kappa p^{1-\kappa}}{\theta^\kappa p^{1-\kappa} + (1-\phi)(1-\theta)^\kappa}, \tag{150}$$

where $\mathfrak{s}_j \equiv \frac{s_j}{\bar{s}}$. The aggregate housing expenditure share ($\bar{e}$) is only constant, given that rents (p) may grow, provided that $\kappa = 1$ (Piazzesi and Schneider 2016). Notice that the utility specification in the main text, given by (2), is the limiting case of the above stated CES utility function for $\kappa \to 1$.

10.3.3 Status Preferences: Multiplicative Reference Level

Status preferences are often also captured as ratios instead of differences (Clark, Frijters, and Shields 2008; Schünemann and Trimborn 2017). A typical formulation looks like this

$$v(c_j, s_j) = \frac{\left[s_j^\theta \left(\frac{s_j}{\bar{s}}\right)^\phi (c_j)^{1-\theta}\right]^{1-\sigma} - 1}{1-\sigma},$$

where $\theta \in (0,1)$, $\phi \in [0,1)$, and $\sigma > 0$. In this case, the housing expenditure share of agent j is given by

$$e_j = \frac{\theta + \phi}{1 + \phi}.$$

Hence, this preference specification is not compatible with heterogenous housing expenditure shares that vary systematically with income.